Hedging Mismatched Currencies with Options and Futures *

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January 2009

JEL classification: F23; F31; D81

Keywords: Options; Futures; Multiple currency risks; Hedging effectiveness

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*We would like to thank seminar participants at the University of Hong Kong for their helpful comments and suggestions. We also thank Ho Yin Yick for his excellent research assistance.

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Abstract

This paper develops an expected utility model of an exporting firm in a developing country where currency derivative markets do not exist. The firm faces exchange rate risk exposure to a foreign currency cash flow. To cross-hedge against this risk exposure, the firm uses currency futures and options between the foreign currency and a third currency. Since a triangular parity condition holds among the three given currencies, this available hedging opportunity, albeit incomplete, is proved to be useful in reducing the firm's exchange rate risk exposure. We show that currency options are redundant under two rather restrictive conditions: (i) logarithmic utility functions and/or (ii) independent spot exchange rates. In a more realistic cross-hedging environment, we show that currency options are optimally used by the firm to incompletely span the missing currency futures between the domestic and foreign currencies. We also estimate the benefits of using currency options for cross-hedging purposes. The improvement in hedging effectiveness can be substantial.

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1. Introduction

Since the collapse of the Bretton Woods Agreement in 1973, exchange rates have become substantially volatile (Meese, 1990), making exchange rate risk management a fact of financial life (Rawls and Smithson, 1990). As documented by Bodnar, Hayt, and Marston (1996, 1998) and Bodnar and Gebhardt (1999), the use of currency forwards, futures, and options is the norm rather than the exception among non-financial corporations.

While currency derivative markets are the hallmark of industrialized countries, they are seldom readily available in less developed countries (LDCs) wherein capital markets are embryonic and foreign exchange markets are heavily controlled. Even if currency forward contracts are available in some LDCs, they are deemed to be forward-cover insurance schemes rather than financial instruments whose prices are freely determined by market forces (Jacque, 1996). Also, in many of the newly industrializing countries of Latin America and Asia Pacific, currency derivative markets are just starting to develop in a rather slow pace (Eiteman, Stonehill, and Moffett, 2001). To indirectly hedge against their exchange rate risk exposure, exporting firms in these countries are obliged to use derivative securities on related currencies. Such an exchange rate risk management technique is referred to as 'cross-hedging.'

The purpose of this paper is to study the hedging decision of an exporting firm in a developing country where currency derivative markets do not exist. We are particularly interested in examining the hedging role of currency options in the context of cross-hedging. To this end, we develop an expected utility model of the exporting firm facing exchange rate uncertainty. There is a third country which has well-developed currency futures and options markets. Since a triangular parity condition holds among the domestic, foreign, and third currencies, this available hedging opportunity, albeit incomplete, remains useful in reducing the firm's exchange rate risk exposure. We show that currency options are redundant under two rather restrictive conditions: (i) logarithmic utility functions and/or (ii) independent spot exchange rates. In a more realistic cross-hedging environment, we show that currency options are optimally used by the firm to incompletely span the missing currency futures between the domestic and foreign currencies. The driving force is the triangular parity condition that gives rise to an exchange rate risk that is multiplicative, thereby non-linear, in nature. This source of non-linearity creates a hedging demand for non-linear currency options, as distinct from that for linear currency futures.

This paper is closest in the spirit of Chang and Wong (2003) who also examine the hedging role of currency options in the context of cross-hedging. The main difference is on how market incompleteness is introduced. In Chang and Wong (2003), no derivative securities exist for the foreign currency but there are currency futures and options between the domestic and third currencies. The degree of incompleteness is less sever in their setting because the hedging instruments are directly related to the domestic currency. The source of non-linearity is shown to be quadratic in nature, which makes the analytical results elegant. In our model, however, the hedging instruments do not involve the domestic currency and the source of non-linearity is of a higher order, thereby complicating the analysis to a great extent.

The rest of the paper is organized as follows. In the next section, we develop an expected utility model of an exporting firm facing multiple currency risks and cross-hedging opportunities. Section 3 derives sufficient conditions under which currency options are never used by the firm. Section 4 shows how the firm can incompletely span the missing currency futures between the domestic and foreign currencies with the available currency options and futures on a third currency. Section 5 offers empirical evidence on the hedging effectiveness of including currency options for cross-hedging purposes. The final section concludes.

2. The model

Consider a one-period, two-date (0 and 1) model of an exporting firm domiciled in a developing country where currency derivative markets do not exist. At date 0, the firm concludes a transaction in a foreign country, which results in a net foreign currency cash flow, X, to be received at date 1. While the size of X is known with certainty *ex ante*, the firm does not know the then prevailing spot exchange rate at date 1, denoted by $\tilde{S}$, which is expressed in units of the domestic currency per unit of the foreign currency.[1] The firm as such encounters exchange rate risk exposure of $\tilde{S}X$.

To hedge its exchange rate risk exposure, the firm has to rely on a related currency of a third country that has well-developed currency derivative markets. Define $\tilde{S}_1$ as the spot exchange rate of the domestic currency against the third currency at date 1. Likewise, define $\tilde{S}_2$ as the spot exchange rate of the third currency against the foreign currency at date 1. Based on these two spot exchange rates, one can derive the cross-rate of the domestic currency against the foreign currency at date 1 as $\tilde{S}_1\tilde{S}_2$. It follows immediately from the Law of One Price that $\tilde{S} = \tilde{S}_1\tilde{S}_2$. Such a triangular parity condition is depicted in Figure 1.

[1] Throughout the paper, random variables have a tilde (~) while their realizations do not.

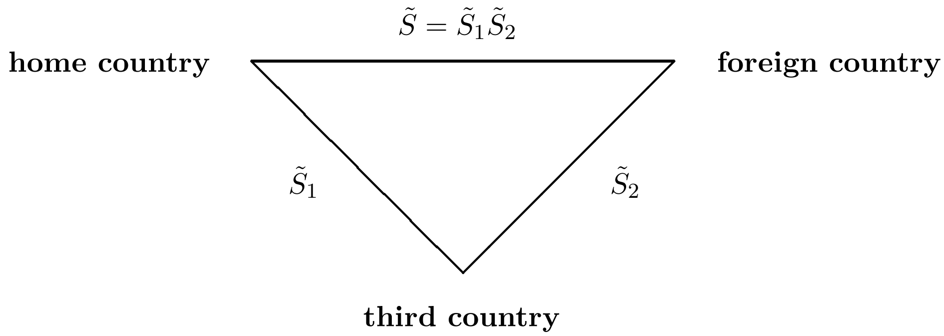

Figure 1. Triangular parity condition

At date 0, the firm can trade infinitely divisible currency futures and options (calls and puts) that call for delivery of the third currency per unit of the foreign currency at date 1. Without any loss of generality, we restrict the firm to use currency futures and put options only.[2] Furthermore, for the pure sake of simplicity, we consider only one strike price, denoted by K, for the currency put options. The strike price, K, is exogenously determined, thereby not a choice variable of the firm.

Let F be the futures price at date 0 and P be the premium on the currency put options, where P is compounded to date 1. To isolate the firm's hedging motive from its speculative motive, it suffices to restrict our attention to the case where the currency futures and put options are perceived as jointly unbiased by the firm. As such, we assume throughout the paper that F equals the expected value of $\tilde{S}_2$ and P equals that of $\max(K - \tilde{S}_2, 0)$.[3]

The firm's date 1 profit denominated in the domestic currency is given by

$$\tilde{\Pi} = \tilde{S}_1\tilde{S}_2 X + \tilde{S}_1(F - \tilde{S}_2)H + \tilde{S}_1[P - \max(K - \tilde{S}_2, 0)]Z, \quad (1)$$

where H and Z are the numbers of the currency futures and put options sold (purchased if negative) by the firm at date 0, respectively. The firm possesses a von Neumann-

[2] Because payoffs of any combinations of futures, calls, and puts can be replicated by any two of these three financial instruments, one of them is redundant.

[3] Our intention here is not to impose an ad hoc option pricing theory but to focus on the hedging role of the put currency options.

Morgenstern utility function, $U(\Pi)$, defined over its domestic currency profit, Π, with $U'(\Pi) > 0$ and $U''(\Pi) < 0$, indicating the presence of risk aversion.[4]

The firm is an expected utility maximizer and has to solve the following *ex ante* decision problem at date 0:

$$\max_{H,Z} \mathrm{E}[U(\tilde{\Pi})], \tag{2}$$

where $\mathrm{E}(\cdot)$ is the expectation operator with respect to the firm's subjective joint probability distribution function of $\tilde{S}_1$ and $\tilde{S}_2$, and $\tilde{\Pi}$ is defined in equation (1). The first-order conditions for program (2) are given by

$$\mathrm{E}[U'(\tilde{\Pi}^*)\tilde{S}_1(F - \tilde{S}_2)] = 0, \tag{3}$$

$$\mathrm{E}\{U'(\tilde{\Pi}^*)\tilde{S}_1[P - \max(K - \tilde{S}_2, 0)]\} = 0, \tag{4}$$

where an asterisk (*) indicates an optimal level. The second-order conditions for a maximum are satisfied given risk aversion.

3. Redundancy of options

Since $F = \mathrm{E}(\tilde{S}_2)$ and $P = \mathrm{E}[\max(K - \tilde{S}_2, 0)]$, we can write equations (3) and (4) as[5]

$$\mathrm{Cov}[U'(\tilde{\Pi}^*)\tilde{S}_1, \tilde{S}_2] = 0, \tag{5}$$

$$\mathrm{Cov}[U'(\tilde{\Pi}^*)\tilde{S}_1, \max(K - \tilde{S}_2, 0)] = 0, \tag{6}$$

[4] For privately held, owner-managed firms, risk-averse behavior prevails. Even for publicly listed firms, managerial risk aversion (Stulz, 1984), corporate taxes (Smith and Stulz, 1985), costs of financial distress (Smith and Stulz, 1985), and capital market imperfections (Stulz, 1990; and Froot, Scharfstein, and Stein, 1993) all imply a concave objective function for firms, thereby justifying the use of risk aversion as an approximation. See Tufano (1996) for evidence that managerial risk aversion is a rationale for corporate risk management in the gold mining industry.

[5] For any two random variables, $\tilde{X}$ and $\tilde{Y}$, $\mathrm{Cov}(\tilde{X}, \tilde{Y}) = \mathrm{E}(\tilde{X}\tilde{Y}) - \mathrm{E}(\tilde{X})\mathrm{E}(\tilde{Y})$.

where $\text{Cov}(\cdot,\cdot)$ is the covariance operator with respect to the firm's subjective joint probability distribution function of $\tilde{S}_1$ and $\tilde{S}_2$. Partially differentiating $\text{E}[U'(\tilde{\Pi}^*)\tilde{S}_1|S_2]$ yields

$$\frac{\partial}{\partial S_2}\text{E}[U'(\tilde{\Pi}^*)\tilde{S}_1|S_2] = \text{E}\bigg\{U'(\tilde{\Pi}^*)[1 - R(\tilde{\Pi}^*)]\frac{\partial \tilde{S}_1}{\partial S_2} + U''(\tilde{\Pi}^*)\tilde{S}_1^2\bigg[X - H^* - \frac{\partial}{\partial S_2}\max(K - S_2, 0)Z^*\bigg]\bigg|S_2\bigg\}, \qquad (7)$$

where $R(\Pi) = -\Pi U''(\Pi)/U'(\Pi)$ is the Arrow-Pratt measure of relative risk aversion. If $\partial\tilde{S}_1/\partial S_2 \equiv 0$ (i.e., $\tilde{S}_1$ and $\tilde{S}_2$ are independent), or if $R(\Pi) \equiv 1$ for all Π (i.e., $U(\Pi)$ is logarithmic), it is evident from equation (7) that $\text{E}[U'(\tilde{\Pi}^*)\tilde{S}_1|S_2]$ would be invariant to different realizations of $\tilde{S}_2$ when $H^* = X$ and $Z^* = 0$ and thus equations (5) and (6) hold simultaneously.

To see the underlying intuition, consider first the case that the firm's utility function, $U(\Pi)$, is logarithmic. In this case, the objective function of program (2) becomes

$$\text{E}(\ln\tilde{\Pi}) = \text{E}(\ln\tilde{S}_1) + \text{E}\bigg\{\ln\{\tilde{S}_2X + (F - \tilde{S}_2)H + [P - \max\,(K - \tilde{S}_2,\ 0)]Z\}\bigg\}.$$

It is evident that $\tilde{S}_1$ does not affect the firm's optimal hedge position and thus the celebrated full-hedging theorem applies (Danthine, 1978; Holthausen, 1979; and Feder, Just, and Schmitz, 1980). Now, consider the case where the two random spot exchange rates, $\tilde{S}_1$ and $\tilde{S}_2$, are independent. When $H = X$ and $Z = 0$, equation (1) implies that $\tilde{\Pi} = \tilde{S}_1FX$. Thus, it follows that equations (5) and (6) hold simultaneously at $H^* = X$ and $Z^* = 0$. In either case, the currency put options are not used by the firm for cross-hedging purposes.

4. Hedging role of options

In the previous section, we have shown that currency options are redundant under logarithmic utility functions and/or independent spot exchange rates. However, it is unduly restrictive to assume either condition to hold. In other words, in a more realistic cross-hedging environment, we would expect currency options to be an integral part of the optimal hedge positions of exporting firms in developing countries.

To verify the above conjecture, let us consider the following simple example. Suppose that $\tilde{S}_1 = \overline{S}_1 + \beta\tilde{\theta}$ and $\tilde{S}_2 = \overline{S}_2 + \tilde{\theta}$, where $\overline{S}_i$ is the expected value of $\tilde{S}_i$ $(i = 1, 2)$, β is a scalar, and $\tilde{\theta}$ is a zero-mean random variable. Thus, $\tilde{S}_1$ and $\tilde{S}_2$ are perfectly negatively (positively) correlated if $\beta < (>)\ 0$. Let $\tilde{\theta}$ take on three possible values: $-\Theta$ with probability p, 0 with probability $1 - 2p$, and Θ with probability p. Then, we have

$$\tilde{S} = \tilde{S}_1\tilde{S}_2 = (\overline{S}_1 + \beta\tilde{\theta})(\overline{S}_2 + \tilde{\theta}) = \overline{S}_1\overline{S}_2 + (\overline{S}_1 + \beta\overline{S}_2)\tilde{\theta} + \beta\tilde{\theta}^2. \tag{8}$$

Given the assumed three-point probability distribution function of $\tilde{\theta}$, we have $\tilde{\theta}^2 = \Theta[\tilde{\theta} + 2\max(-\tilde{\theta}, 0)]$. Thus, using this fact and equation (8), the exchange rate risk exposure faced by the firm is given by

$$\tilde{S}X = \overline{S}_1\overline{S}_2X + (\overline{S}_1 + \beta\overline{S}_2 + \beta\Theta)\tilde{\theta}X + 2\beta\Theta\max(-\tilde{\theta}, 0)X. \tag{9}$$

Let $K = F$. Since F equals the expected value of $\tilde{S}_2$, we have $F = K = \overline{S}_2$. Then, the payoff of a hedge position, (H, Z), is given by

$$\tilde{V} = (\overline{S}_1 + \beta\tilde{\theta})\{\tilde{\theta}H + [\max(-\tilde{\theta}, 0) - P]Z\}. \tag{10}$$

Given the assumed three-point probability distribution function of $\tilde{\theta}$, we have $\tilde{\theta}^2 = \Theta[\tilde{\theta} + 2\max(-\tilde{\theta}, 0)]$ and $\tilde{\theta}\max(-\tilde{\theta}, 0) = -\Theta\max(-\tilde{\theta}, 0)$. Thus, equation (10) can be written as

$$\tilde{V} = [(\overline{S}_1 + \beta\Theta)H - \beta PZ]\tilde{\theta} + [2\beta\Theta H + (\overline{S}_1 - \beta\Theta)Z]\max(-\tilde{\theta}, 0) - \overline{S}_1PZ. \tag{11}$$

Inspection of equations (9) and (11) reveals that the firm's exchange rate risk exposure can be completely eliminated by the hedge position, (H^*, Z^*), which satisfies

$$[\overline{S}_1 + \beta\overline{S}_2 + \beta\Theta]X = (\overline{S}_1 + \beta\Theta)H^* - \beta PZ^*, \tag{12}$$

$$2\beta\Theta X = 2\beta\Theta H^* + (\overline{S}_1 - \beta\Theta)Z^*. \tag{13}$$

Solving equations (12) and (13) yields

$$H^* = X - \frac{\overline{S}_1 - \beta\Theta}{2\beta\Theta} Z^*,$$

$$Z^* = -\frac{2\beta^2 \overline{S}_2 \Theta X}{(\overline{S}_1 - \beta\Theta)(\overline{S}_1 + \beta\Theta) + 2\beta^2 P\Theta}.$$

As long as $\beta \neq 0$, we have $Z^* < 0$. Thus, in order to synthesize a short position of the missing currency futures between the domestic and foreign currencies, a long position of the currency put options between the foreign and third currencies is called for irrespective of whether $\tilde{S}_1$ and $\tilde{S}_2$ are negatively or positively correlated.

In general, complete spanning of the missing currency futures contracts between the domestic and foreign currencies is not feasible. To facilitate the empirical study of hedging effectiveness in the next section, we hereafter restrict our attention to the case where the firm's objective is to minimize the variance of its date 1 domestic currency profit:

$$\min_{H,Z} \ \mathrm{Var}(\tilde{\Pi}) = \mathrm{E}\{[\tilde{\Pi} - \mathrm{E}(\tilde{\Pi})]^2\}, \tag{14}$$

where $\mathrm{Var}(\cdot)$ is the variance operator with respect to the firm's subjective joint probability distribution function of $\tilde{S}_1$ and $\tilde{S}_2$, and $\tilde{\Pi}$ is defined in equation (1).

The first-order conditions for program (14) are given by

$$\mathrm{Cov}[\tilde{\Pi}^*, \tilde{S}_1(F - \tilde{S}_2)] = 0, \tag{15}$$

$$\mathrm{Cov}\{\tilde{\Pi}^*, \tilde{S}_1[P - \max(K - \tilde{S}_2, 0)]\} = 0. \tag{16}$$

where an asterisk (*) indicates an optimal level.[6] Solving equations (15) and (16) yields

$$\begin{pmatrix} H^* \\ Z^* \end{pmatrix} = -\mathbf{A}^{-1}\mathbf{B}X, \tag{17}$$

[6]The second-order conditions for a minimum are satisfied.

where

$$\mathbf{A} = \begin{pmatrix} \mathrm{Var}[\tilde{S}_1(F-\tilde{S}_2)] & \mathrm{Cov}\{\tilde{S}_1(F-\tilde{S}_2), \tilde{S}_1[P-\max(K-\tilde{S}_2,0)]\} \\ \mathrm{Cov}\{\tilde{S}_1(F-\tilde{S}_2), \tilde{S}_1[P-\max(K-\tilde{S}_2,0)]\} & \mathrm{Var}\{\tilde{S}_1[P-\max(K-\tilde{S}_2,0)]\} \end{pmatrix},$$

$$\mathbf{B} = \begin{pmatrix} \mathrm{Cov}[\tilde{S}_1\tilde{S}_2, \tilde{S}_1(F-\tilde{S}_2)] \\ \mathrm{Cov}\{\tilde{S}_1\tilde{S}_2, \tilde{S}_1[P-\max(K-\tilde{S}_2,0)]\} \end{pmatrix}.$$

Inspection of equation (17) reveals that Z^* is in general non-zero. However, without specifying the underlying joint probability distribution function of $\tilde{S}_1$ and $\tilde{S}_2$, the sign of Z^* is a priori indeterminate.

When the currency put options are absent (i.e., $Z \equiv 0$), it is easily shown that the firm's optimal futures position, H^0, is given by

$$H^0 = -\frac{\mathrm{Cov}[\tilde{S}_1\tilde{S}_2, \tilde{S}_1(F-\tilde{S}_2)]}{\mathrm{Var}[\tilde{S}_1(F-\tilde{S}_2)]}. \tag{18}$$

The sign of H^0 is opposite to that of $\mathrm{Cov}[\tilde{S}_1\tilde{S}_2, \tilde{S}_1(F-\tilde{S}_2)]$, which is also a priori indeterminate without knowing the functional form of the joint probability distribution function of $\tilde{S}_1$ and $\tilde{S}_2$.

5. Hedging effectiveness

To empirically evaluate the potential usefulness of cross-hedging with currency options and futures, we assume that $\tilde{S}_1$ and $\tilde{S}_2$ are jointly log-normally distributed:

$$\begin{pmatrix} \ln\tilde{S}_1 \\ \ln\tilde{S}_2 \end{pmatrix} \sim N\left[\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}\right].$$

Alternatively, we can write $\tilde{S}_1 = \exp(\mu_1 + \sigma_1\tilde{\varepsilon}_1)$ and $\tilde{S}_2 = \exp(\mu_2 + \sigma_2\tilde{\varepsilon}_2)$, where $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ are both standard normal random variables with $\mathrm{Cov}(\tilde{\varepsilon}_1, \tilde{\varepsilon}_2) = \rho$. In Appendix A, we prove the following useful result:

$$\mathrm{E}\Big(\tilde{S}_1^m \tilde{S}_2^n I_{\{\tilde{S}_2<K\}}\Big) = \exp\left(m\mu_1 + n\mu_2 + \frac{m^2\sigma_1^2}{2} + mn\rho\sigma_1\sigma_2 + \frac{n^2\sigma_2^2}{2}\right)$$
$$\times\ \Phi\Big(\frac{\ln K - \mu_2}{\sigma_2} - m\rho\sigma_1 - n\sigma_2\Big), \tag{19}$$

where $I_{\{\tilde{S}_2<K\}}$ is the indicator function for the event that $\tilde{S}_2 < K$, and $\Phi(\cdot)$ is the cumulative standard normal distribution function.

Equation (19) can be used repeatedly to express all the moments in equation (17) as functions of the underlying distribution parameters. For example, by setting $m = 0$, $n = 1$, and $K = \infty$ in equation (19), we have

$$F = \mathrm{E}(\tilde{S}_2) = \exp\left(\mu_2 + \frac{\sigma_2^2}{2}\right). \tag{20}$$

Note that

$$P = \mathrm{E}[\max(K - \tilde{S}_2, 0)] = \mathrm{E}\Big[(K - \tilde{S}_2)I_{\{\tilde{S}_2<K\}}\Big] = K\mathrm{E}\Big(I_{\{\tilde{S}_2<K\}}\Big) - \mathrm{E}\Big(\tilde{S}_2 I_{\{\tilde{S}_2<K\}}\Big).$$

The first term on the right-hand side of the above equation corresponds to equation (19) when $m = n = 0$ and the second term to $m = 0$ and $n = 1$. Thus, we have

$$P = K\Phi\Big(\frac{\ln K - \mu_2}{\sigma_2}\Big) - \mathrm{E}(\tilde{S}_2)\Phi\Big(\frac{\ln K - \mu_2}{\sigma_2} - \sigma_2\Big).$$

When $K = \mathrm{E}(\tilde{S}_2)$, the above equation can be further reduced to

$$P = \exp\left(\mu_2 + \frac{\sigma_2^2}{2}\right)\Big[1 - 2\Phi\Big(-\frac{\sigma_2}{2}\Big)\Big]. \tag{21}$$

In Appendix B, we report all the variances and covariances contained in equation (17) in terms of the underlying distribution parameters. It is interesting to observe that μ_1 and μ_2, albeit essential for all the relevant moments, do not appear in the ultimate optimal hedge ratios. As such, no information on the means of $\ln \tilde{S}_1$ and $\ln \tilde{S}_2$ is needed for finding the optimal hedge ratios.

The empirical exercise is as follows. We refer to Indonesia as the home country, Taiwan as the foreign country, and the United States as the third country. Thus, $\tilde{S}_1$ is the random spot exchange rate for the Indonesian rupee against the US dollar and $\tilde{S}_2$ is that for the US dollar against the Taiwanese dollar. The foreign currency cash flow, X, is normalized to one unit of the foreign currency so that the firm's hedge position is simply the hedge ratio. The prices of the currency futures and put options with $K = F$ are artificially set equal to equations (20) and (21), respectively. Daily data on spot exchange rates are extracted from *Datastream* for the period from January 1, 1997 to December 31, 2001.

In each year, we substitute the sample variances of $\ln \tilde{S}_1$ and $\ln \tilde{S}_2$ and the sample covariance $\ln \tilde{S}_1$ and $\ln \tilde{S}_1$ into equations (17) and (18), using the results in Appendix B, to compute the firm's optimal cross-hedge ratios, (H^*, Z^*) and H^0, respectively. These results are reported in Table 1.

Table 1. Data Description

	1997	1998	1999	2000	2001
Observations	261	261	261	260	261
US Dollar/Taiwanese Dollar					
Mean ($\times 10^{-2}$)	3.49	2.99	3.10	3.21	2.96
Standard Deviation ($\times 10^{-3}$)	1.84	0.81	0.49	0.78	0.84
US Dollar/Indonesian Rupee					
Mean ($\times 10^{-4}$)	3.64	1.04	1.29	1.20	0.98
Standard Deviation ($\times 10^{-5}$)	6.95	2.38	1.28	1.10	0.79
Optimal Cross-Hedge Ratios					
H^0	−2.558	−5.437	−2.583	−1.591	1.020
H^*	−3.342	−6.907	−2.760	−1.714	1.019
Z^*	−1.243	−2.352	−0.329	−0.229	−0.001

This table presents descriptive statistics on the two spot exchange rate series from January 1, 1997 to December 31, 2001. Optimal cross-hedge ratios are computed from equations (17) and (18) in the text.

To compare the performances of using currency options and futures for cross-hedging purposes, we need to estimate the variances of the firm's date 1 domestic currency profit:

$$\mathrm{Var}\{\tilde{S}_1\tilde{S}_2X + \tilde{S}_1(F - \tilde{S}_2)H + \tilde{S}_1[P - \max(K - \tilde{S}_2, 0)]Z\},$$

for three cases: (i) no hedging (i.e., $H = Z = 0$), (ii) hedging with futures only (i.e., $H = H^0$ and $Z = 0$), and (iii) hedging with options and futures(i.e., $H = H^*$ and $Z = Z^*$). To this end, we compute, each year, three sample variances of the firm's date 1 domestic currency profit using the actual realized spot exchange rates during the year. The first sample variance is based on $H = Z = 0$. The second sample variance is based on $H = H^0$ and $Z = 0$, where H^0 is taken from Table 1. The third sample variance is based on $H = H^*$ and $Z = Z^*$, where $(H^*,\ Z^*)$ is taken from Table 1. These results are reported in Table 2.

Table 2. Comparisons of Hedging Effectiveness

	1997	1998	1999	2000	2001
Portfolio Variances					
No Hedging	407	4306	480	398	657
Hedging with Futures only	90.1	1874	381	300	590
Hedging with Futures and Options	81.4	1881	383	295	590
Variance Reductions of Hedging with Options and Futures Compared to:					
No Hedging	80.0%	56.3%	20.2%	25.9%	10.2%
Hedging with Futures only	9.65%	−0.36%	−0.39%	1.43%	0%

This table shows the portfolio variances for three cases: (i) no hedging (i.e., $H = Z = 0$), (ii) hedging with futures only (i.e., $H = H^0$ and $Z = 0$), and (iii) hedging with futures and options (i.e., $H = H^*$ and $Z = Z^*$). The percentage variance reductions of hedging with options and futures over no hedging and hedging with futures only are also given.

As shown in Table 2, using currency options and futures for cross-hedging purposes on average results in variance reductions of about 39% as compared to the no hedging strategy, and about 2% as compared to using currency futures as the sole hedging instrument. Although including currency options in hedge positions may sometimes lower the hedging effectiveness as compared to using currency futures alone, the increase in portfolio variance is almost negligible (less than 0.4%). The potential improvement in hedging effectiveness, however, can be quite substantial (more than 9%). Hence, exporting firms in developing countries should find currency options useful for cross-hedging purposes.

6. Conclusion

In the post-Bretton Woods era, exchange rates have been increasingly volatile, making non-financial firms take exchange rate risk management very seriously. This paper has studied how a risk-averse exporting firm, confronting a foreign currency cash flow but possessing no direct hedging opportunities, can employ derivative securities on a related currency to reduce its exchange rate risk exposure. Currency options play no role as a hedging instrument under two rather restrictive conditions of logarithmic utility functions and/or independent spot exchange rates. In a more realistic cross-hedging environment, we have shown how the firm can use currency options to incompletely replicate the missing risk-sharing contract. The driving force is a triangular parity condition which gives rise to an exchange rate risk that is multiplicative, thereby non-linear, in nature. This source of non-linearity creates a hedging demand for non-linear currency options, as distinct from that for linear currency futures.

Appendix

A. Derivation of equation (19)

Using the fact that $\tilde{S}_i = \exp(\mu_i + \sigma_i \tilde{\varepsilon}_i)$, $i = 1$ and 2, we have

$$\mathrm{E}\Big(\tilde{S}_1^m \tilde{S}_2^n I_{\{\tilde{S}_2 < K\}}\Big) = \frac{1}{2\pi\sqrt{1-\rho^2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\frac{\ln K - \mu_2}{\sigma_2}} \exp[m(\mu_1 + \sigma_1\varepsilon_1)] \exp[n(\mu_2 + \sigma_2\varepsilon_2)]$$

$$\times \ \exp\left[-\frac{\varepsilon_1^2 + \varepsilon_2^2 - 2\rho\varepsilon_1\varepsilon_2}{2(1-\rho^2)}\right] \mathrm{d}\varepsilon_1 \ \mathrm{d}\varepsilon_2$$

$$= \frac{1}{2\pi\sqrt{1-\rho^2}} \int_{-\infty}^{\infty} \exp\left[m(\mu_1 + \sigma_1\varepsilon_1) - \frac{\varepsilon_1^2}{2(1-\rho^2)}\right]$$

$$\times \left\{ \int_{-\infty}^{\frac{\ln K - \mu_2}{\sigma_2}} \exp[n(\mu_2 + \sigma_2\varepsilon_2)] \exp\left[-\frac{\varepsilon_2^2 - 2\rho\varepsilon_1\varepsilon_2}{2(1-\rho^2)}\right] \mathrm{d}\varepsilon_2 \right\} \mathrm{d}\varepsilon_1$$

$$= \frac{\exp(m\mu_1 + n\mu_2)}{2\pi\sqrt{1-\rho^2}} \int_{-\infty}^{\infty} \exp\left\{ m\sigma_1\varepsilon_1 + \frac{[\rho\varepsilon_1 + (1-\rho^2)n\sigma_2]^2 - \varepsilon_1^2}{2(1-\rho^2)} \right\}$$

$$\times \left\{ \int_{-\infty}^{\frac{\ln K - \mu_2}{\sigma_2}} \exp\left\{ -\frac{[\varepsilon_2 - \rho\varepsilon_1 - (1-\rho^2)n\sigma_2]^2}{2(1-\rho^2)} \right\} \mathrm{d}\varepsilon_2 \right\} \mathrm{d}\varepsilon_1$$

$$= \frac{1}{\sqrt{2\pi}} \exp\left(m\mu_1 + n\mu_2 + \frac{m^2\sigma_1^2}{2} + mn\rho\sigma_1\sigma_2 + \frac{n^2\sigma_2^2}{2} \right)$$

$$\times \int_{-\infty}^{\infty} \exp\left[-\frac{(\varepsilon_1 - m\sigma_1 - n\rho\sigma_2)^2}{2} \right] \Phi\left[\frac{\frac{\ln K - \mu_2}{\sigma_2} - \rho\varepsilon_1 - (1-\rho^2)n\sigma_2}{\sqrt{1-\rho^2}} \right] \mathrm{d}\varepsilon_1$$

$$= \frac{1}{\sqrt{2\pi}} \exp\left(m\mu_1 + n\mu_2 + \frac{m^2\sigma_1^2}{2} + mn\rho\sigma_1\sigma_2 + \frac{n^2\sigma_2^2}{2} \right)$$

$$\times \int_{-\infty}^{\infty} \exp\left(-\frac{z^2}{2} \right) \Phi\left(\frac{\frac{\ln K - \mu_2}{\sigma_2} - \rho z - m\rho\sigma_1 - n\sigma_2}{\sqrt{1-\rho^2}} \right) \mathrm{d}z$$

$$= \frac{1}{\sqrt{2\pi}} \exp\left(m\mu_1 + n\mu_2 + \frac{m^2\sigma_1^2}{2} + mn\rho\sigma_1\sigma_2 + \frac{n^2\sigma_2^2}{2} \right)$$

$$\times \; \mathrm{E}_z\Bigg[\Phi\Bigg(\frac{\frac{\ln K-\mu_2}{\sigma_2}-\rho z-m\rho\sigma_1-n\sigma_2}{\sqrt{1-\rho^2}}\Bigg)\Bigg], \tag{A-1}$$

where $z=\varepsilon_1-m\sigma_1-n\rho\sigma_2$ is a standard normal random variable and $\mathrm{E}_z(\cdot)$ is the expectation operator with respect to $\Phi(z)$. Lemma 1 in Lien (1986) states that

$$\mathrm{E}_z[\Phi(\alpha+\beta z)]=\Phi\Bigg(\frac{\alpha}{\sqrt{1+\beta^2}}\Bigg). \tag{A-2}$$

Using equation (A-2) by setting $\alpha=\frac{\ln K-\mu_2}{\sigma_2}-m\rho\sigma_1-n\sigma_2$ and $\beta=\frac{\rho}{\sqrt{1-\rho^2}}$, equation (A-1) reduces to equation (19).

B. *Derivation of variances and covariances*

Note that $\mathrm{Var}[\tilde{S}_1(F-\tilde{S}_2)]=\mathrm{E}[\tilde{S}_1^2(F-\tilde{S}_2)^2]-\mathrm{E}[\tilde{S}_1(F-\tilde{S}_2)]^2$, which can be written as

$$\mathrm{E}(\tilde{S}_1^2)F^2-2\mathrm{E}(\tilde{S}_1^2\tilde{S}_2)F+\mathrm{E}(\tilde{S}_1^2\tilde{S}_2^2)-\mathrm{E}(\tilde{S}_1)^2F^2+2\mathrm{E}(\tilde{S}_1)\mathrm{E}(\tilde{S}_1\tilde{S}_2)F-\mathrm{E}(\tilde{S}_1\tilde{S}_2)^2.$$

Using equation (19) repeatedly, we have

$$\mathrm{Var}[\tilde{S}_1(F-\tilde{S}_2)]=M\times\;[\exp(\sigma_1^2)-2\exp(\sigma_1^2+2\rho\sigma_1\sigma_2)+\exp(\sigma_1^2+4\rho\sigma_1\sigma_2+\sigma_2^2)$$

$$-1+2\exp(\rho\sigma_1\sigma_2)-\exp(2\rho\sigma_1\sigma_2)],$$

where $M=\exp(2\mu_1+2\mu_2+\sigma_1^2+\sigma_2^2)$. Similarly, we can show that

$$\mathrm{Cov}[\tilde{S}_1\tilde{S}_2,\tilde{S}_1(F-\tilde{S}_2)]=M\times\;[\exp(\sigma_1^2+2\rho\sigma_1\sigma_2)-\exp(\sigma_1^2+4\rho\sigma_1\sigma_2+\sigma_2^2)$$

$$-\exp(\rho\sigma_1\sigma_2)+\exp(2\rho\sigma_1\sigma_2)],$$

$$\mathrm{Var}\{\tilde{S}_1[P-\max(K-\tilde{S}_2,0)]\}=M\times\;\Bigg\{\exp(\sigma_1^2)\Bigg[2\Phi\Bigg(\frac{\sigma_2}{2}\Bigg)-1\Bigg]\Bigg[2\Phi\Bigg(\frac{\sigma_2}{2}\Bigg)-1-2\Phi\Bigg(\frac{\sigma_2}{2}-2\rho\sigma_1\Bigg)\Bigg]$$

$$+4\exp(\sigma_1^2+2\rho\sigma_1\sigma_2)\Bigg[\Phi\Bigg(\frac{\sigma_2}{2}\Bigg)-1\Bigg]\Phi\Bigg(-\frac{\sigma_2}{2}-2\rho\sigma_1\Bigg)$$

$$+\exp(\sigma_1^2)\Phi\left(\frac{\sigma_2}{2}-2\rho\sigma_1\right)+\exp(\sigma_1^2+4\rho\sigma_1\sigma_2+\sigma_2^2)\Phi\left(-\frac{3\sigma_2}{2}-2\rho\sigma_1\right)$$

$$-\left[2\Phi\left(\frac{\sigma_2}{2}\right)-1-\Phi\left(\frac{\sigma_2}{2}-\rho\sigma_1\right)+\exp(\rho\sigma_1\sigma_2)\Phi\left(-\frac{\sigma_2}{2}-\rho\sigma_1\right)\right]^2\Bigg\},$$

$$\mathrm{Cov}\{\tilde{S}_1(F-\tilde{S}_2),\tilde{S}_1[P-\max(K-\tilde{S}_2,0)]\}=M\times\ \Bigg\{[\exp(\sigma_1^2)-\exp(\sigma_1^2+2\rho\sigma_1\sigma_2)]\left[2\Phi\left(\frac{\sigma_2}{2}\right)-1\right]$$

$$-\exp(\sigma_1^2)\Phi\left(\frac{\sigma_2}{2}-2\rho\sigma_1\right)+2\exp(\sigma_1^2+2\rho\sigma_1\sigma_2)\Phi\left(-\frac{\sigma_2}{2}-2\rho\sigma_1\right)$$

$$-\exp(\sigma_1^2+4\rho\sigma_1\sigma_2+\sigma_2^2)\Phi\left(-\frac{3\sigma_2}{2}-2\rho\sigma_1\right)-[1-\exp(\rho\sigma_1\sigma_2)]$$

$$\times\left[1-2\Phi\left(-\frac{\sigma_2}{2}\right)-\Phi\left(\frac{\sigma_2}{2}-\rho\sigma_1\right)+\exp(\rho\sigma_1\sigma_2)\Phi\left(-\frac{\sigma_2}{2}-\rho\sigma_1\right)\right]\Bigg\},$$

$$\mathrm{Cov}\{\tilde{S}_1\tilde{S}_2,\tilde{S}_1[P-\max(K-\tilde{S}_2,0)]\}=M\times\ \Bigg\{[\exp(\sigma_1^2+2\rho\sigma_1\sigma_2)-\exp(\rho\sigma_1\sigma_2)]\left[2\Phi\left(\frac{\sigma_2}{2}\right)-1\right]$$

$$-\exp(\sigma_1^2+2\rho\sigma_1\sigma_2)\Phi\left(-\frac{\sigma_2}{2}-2\rho\sigma_1\right)-\exp(2\rho\sigma_1\sigma_2)\Phi\left(-\frac{\sigma_2}{2}-\rho\sigma_1\right)\Bigg\}$$

$$+\exp(\sigma_1^2+4\rho\sigma_1\sigma_2+\sigma_2^2)\Phi\left(-\frac{3\sigma_2}{2}-2\rho\sigma_1\right)+\exp(\rho\sigma_1\sigma_2)\Phi\left(\frac{\sigma_2}{2}-\rho\sigma_1\right),$$

$$\mathrm{Var}(\tilde{S}_1S_2)=M\times\ [\exp(\sigma_1^2+4\rho\sigma_1\sigma_2+\sigma_2^2)-\exp(2\rho\sigma_1\sigma_2)].$$

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