Cross-Hedging for the Multinational Firm under Exchange Rate Uncertainty [*]

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This paper examines the impact of cross-hedging on the behavior of the risk-averse multinational firm (MNF) under multiple sources of exchange rate uncertainty. The MNF has operations domiciled in the home country and in a foreign country, each of which produces a single homogeneous good to be sold in the home and foreign markets. Since there are no currency derivative markets between the domestic and foreign currencies, the MNF has to rely on a currency futures market in a third country to cross-hedge its exchange rate risk exposure. We show that the MNF optimally opts for an under-hedge, sells less (more) and produces more (less) in the foreign (home) country. When the set of hedging instruments made available to the MNF is expanded to include currency option contracts between the domestic and third currencies, we show that the MNF optimally opts for a long option position. This paper thus offers a rationale for the cross-hedging role of currency options for MNFs under multiple sources of exchange rate uncertainty.

JEL classification: D81; F23; F31

Keywords: Cross-hedging; Multinationals; Prudence

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Abstract

This paper examines the impact of cross-hedging on the behavior of the risk-averse multinational firm (MNF) under multiple sources of exchange rate uncertainty. The MNF has operations domiciled in the home country and in a foreign country, each of which produces a single homogeneous good to be sold in the home and foreign markets. Since there are no currency derivative markets between the domestic and foreign currencies, the MNF has to rely on a currency futures market in a third country to cross-hedge its exchange rate risk exposure. We show that the MNF optimally opts for an under-hedge, sells less (more) and produces more (less) in the foreign (home) country. When the set of hedging instruments made available to the MNF is expanded to include currency option contracts between the domestic and third currencies, we show that the MNF optimally opts for a long option position. This paper thus offers a rationale for the cross-hedging role of currency options for MNFs under multiple sources of exchange rate uncertainty.

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1. Introduction

Since the collapse of the Bretton Woods Agreement in 1973, foreign exchange rates have become substantially volatile (Meese, 1990). In response to increased exchange rate fluctuations and to various accounting rules and regulations, corporations that source and sell abroad have to devote themselves to devising ways that reduce their exchange rate risk exposure.[1] This is evident from a survey conducted by Rawls and Smithson (1990) that

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[1]The Financial Accounting Standards Board requires transaction and translation gains and losses to

exchange rate risk management is ranked as one of the primary corporate objectives. As documented by Bodnar, Hayt, and Marston (1998), the use of currency forwards, futures, and options is the norm rather than the exception in modern corporations.

While currency derivative markets for forwards, futures, and options are the hallmark of industrialized countries, they are seldom readily available in less developed countries in which capital markets are embryonic and foreign exchange markets are heavily controlled.[2] Also, in many of the newly industrializing countries of Latin America and Asia Pacific, currency derivative markets are just starting to develop in a rather slow pace.[3] As such, international firms that are exposed to currencies of these countries have to avail themselves of derivative securities on related currencies to cross-hedge their exchange rate risk exposure (Eaker and Grant, 1987; Broll and Eckwert, 1996; Broll, 1997; Broll and Wong, 1999; Broll, Wong, and Zilcha, 1999; Chang and Wong, 2003; and Wong, 2007a). Since currencies for which no organized derivative markets exist are by and large currencies of countries with poorly developed capital markets, alternative hedging methods such as borrowing or lending in those currencies are either limited or not available, making cross-hedging a rather unique way of managing risk exposure to those currencies.

The purpose of this paper is to study the impact of cross-hedging on the behavior of the risk-averse multinational firm (MNF) under exchange rate uncertainty. To this end, we develop a variant model of the MNF under multiple sources of exchange rate uncertainty along the lines of Broll and Zilcha (1992), Lien and Wong (2005), Meng and Wong (2007), and Wong (2007b). The MNF has operations domiciled in the home country and in a foreign country. Each of these two operations produces a single homogeneous good to be sold in the home and foreign markets. There are no currency derivative markets between the domestic and foreign currencies. There is, however, a currency futures market between the domestic

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be reported in income statements. The Securities and Exchange Commission, on the other hand, requires statements explaining the impact of exchange rate movements on reported profits to be included in annual reports.

[2]In some less developed countries, currency forward contracts, albeit available, are deemed to be forward-cover insurance schemes rather than financial instruments whose prices are freely determined by market forces. See Jacque (1996).

[3]See Eiteman, Stonehill, and Moffett (2004) for a description of the currency regime and the status of currency derivatives in many of these so-called "exotic currencies."

currency and a third currency to which the MNF has access. Since a triangular parity condition holds among these three currencies, cross-hedging provided by the available, yet incomplete, currency futures market is useful to the MNF in reducing its exchange rate risk exposure.

In the benchmark case of perfect hedging in which the MNF has access to a currency futures market between the domestic and foreign currencies, the celebrated separation and full-hedging theorems in the literature on the MNF under exchange rate uncertainty hold (see, e.g., Broll, 1992; Adam-Müller, 1997; Broll and Wong, 1999, 2006; Broll, Wong, and Zilcha, 1999; and Wong, 2006). The separation theorem states that the MNF's production and sales decisions depend neither on its risk attitude nor on the underlying exchange rate uncertainty. The full-hedging theorem states that the MNF should completely eliminate its exchange rate risk exposure by adopting a full-hedge via the unbiased currency futures contracts.

In the case of cross-hedging in which only the currency futures market between the domestic and third currencies exists, we show that the MNF optimally opts for an under-hedge should it be prudent in the sense of Kimball (1990, 1993).[4] The MNF's sales in the foreign market as such are embedded with residual exchange rate risk that cannot be hedged via the currency futures contracts. We show that the MNF demands a positive risk premium on its foreign sales. This creates a wedge between the marginal revenues in the home and foreign markets. In response to a lack of perfect hedging, the MNF optimally sells less (more) and produces more (less) in the foreign (home) country, in accord with the findings of Broll and Zilcha (1992), Lien and Wong (2005), Meng and Wong (2007), and Wong (2007b). These adjustments in sales and outputs result in a lower expected global domestic currency profit accrued to and a lower expected utility level attainable by the MNF, as compared to those in the benchmark case of perfect hedging. The under-hedging result is consistent with the normal practice of most companies that do not use financial

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[4]Loosely speaking, prudence refers to the propensity to prepare and forearm oneself under uncertainty, in contrast to risk aversion that is how much one dislikes uncertainty and would turn away from it if one could.

derivatives to completely hedge their risk exposure (Tufano, 1996; and Bodnar, Hayt, and Marston, 1998).

We further expand the set of hedging instruments made available to the MNF to include currency option contracts between the domestic and third currencies. Since the triangular parity condition holds among the domestic, foreign, and third currencies, the multiple sources of exchange rate uncertainty faced by the MNF are multiplicative, thereby non-linear, in nature. Such a non-linearity creates a hedging demand for non-linear payoff currency options distinct from that for linear payoff currency futures. We show that the prudent MNF optimally opts for a long option position for cross-hedging purposes, a result consistent with the prevalent use of currency options by non-financial firms (Bodnar, Hayt, and Marston, 1998).

Since the seminal paper of Anderson and Danthine (1981), cross-hedging has become an important strand of the hedging literature. Briys, Crouhy, and Schlesinger (1993), Broll and Eckwert (1996), Adam-Müller (1997), and Broll, Wong, and Zilcha (1999) look at the case where the multiple sources of uncertainty are independent. Broll and Wong (1999), Chang and Wong (2003), and Wong (2003b) allow linear dependence structure to accommodate correlated sources of uncertainty. Wong (2007a) introduces general non-linear dependence structure that enables intuitive prescriptions of cross-hedging strategies for exporting firms. While this paper focuses on the case of independent sources of exchange rate uncertainty, our analysis can be readily applied to the case of more general dependence structure as in Broll and Wong (1999), Chang and Wong (2003), and Wong (2003b, 2007a).

This paper is also related to the burgeoning literature on the hedging role of options. Moschini and Lapan (1992), Wong (2003a), and Adam-Müller and Wong (2006) show that export flexibility leads to an ex-ante profit function that is convex in prices. This in-duced convexity makes options a useful hedging instrument. Sakong, Hayes, and Hallam (1993), Moschini and Lapan (1995), Brown and Toft (2002), Chang and Wong (2003), Wong (2003b), and Lien and Wong (2004) show that firms facing both hedgeable and non-hedgeable risks would optimally use options for hedging purposes. The hedging demand for

options in this case arises from the fact that the two sources of uncertainty interact in a multiplicative manner, which affects the curvature of profit functions. Lence, Sakong, and Hayes (1994) show that forward-looking firms use options for dynamic hedging purposes because they care about the effects of future output prices on profits from future production cycles. Frechette (2001) demonstrates the value of options in a hedge portfolio when there are transaction costs, even though markets themselves may be unbiased. Futures and options are shown to be highly substitutable and the optimal mix of them is rarely one-sided. Broll, Chow, and Wong (2001) document the existence of a non-linear component in the spot-futures exchange rate relationship in five out of six currencies of developed countries. Since the underlying uncertainty is non-linear in nature, options are needed to achieve better hedging performance. Lien and Wong (2002) justify the hedging role of options with multiple delivery specifications in futures markets. The presence of delivery risk creates a truncation of the price distribution, thereby calling for the use of options as a hedging instrument. In a similar vein, Wong and Xu (2006), Meng and Wong (2007), and Wong (2007b) offer another rationale for the hedging role of options when liquidity risk prevails.

The rest of this paper is organized as follows. Section 2 delineates our model of the MNF facing multiple sources of exchange rate uncertainty. Section 3 presents the benchmark case of perfect hedging. Section 4 examines the impact of cross-hedging on the MNF's optimal hedging and sales decisions. Section 5 introduces currency option contracts to the MNF and establishes the hedging role of options. The final section concludes.

2. The model

Consider a one-period, two-date (0 and 1) model of the multinational firm (MNF) under exchange rate uncertainty *à la* Broll and Zilcha (1992), Lien and Wong (2005), Meng and Wong (2007), and Wong (2007b). The MNF has operations domiciled in the home country and in a foreign country. To simplify notation, we suppress the known riskless rate of

interest by compounding all cash flows to their future values at date 1.

The MNF's home operation produces a single homogeneous good, x, according to a cost function, $c_x(x)$, denominated in the domestic currency. We assume that $c_x(x)$ is strictly increasing and convex to reflect the fact that the MNF's production technology exhibits decreasing returns to scale. At date 1, the MNF sells x_h and x_f units of the good, x, in the home and foreign countries, respectively. The sales in the home market generate a revenue function, $r_x(x_h)$, denominated in the domestic currency, whereas the sales in the foreign market generate a revenue function, $R_x(x_f)$, denominated in the foreign currency. We assume that $r_x(x_h)$ and $R_x(x_f)$ are strictly increasing and concave to capture the idea that the MNF is likely to enjoy some monopoly power in the home and foreign markets.

The MNF's foreign operation produces another single homogeneous good, y, according to a cost function, $c_y(y)$, denominated in the foreign currency, where $c_y(y)$ is strictly increasing and convex. We assume that the two homogeneous goods, x and y, are independent of each other.[5] At date 1, the MNF sells y_h and y_f units of the good, y, in the home and foreign countries, respectively. The sales in the home market generate a revenue function, $r_y(y_h)$, denominated in the domestic currency, whereas the sales in the foreign market generate a revenue function, $R_y(y_f)$, denominated in the foreign currency. The revenue functions, $r_y(y_h)$ and $R_y(y_f)$, are strictly increasing and concave.
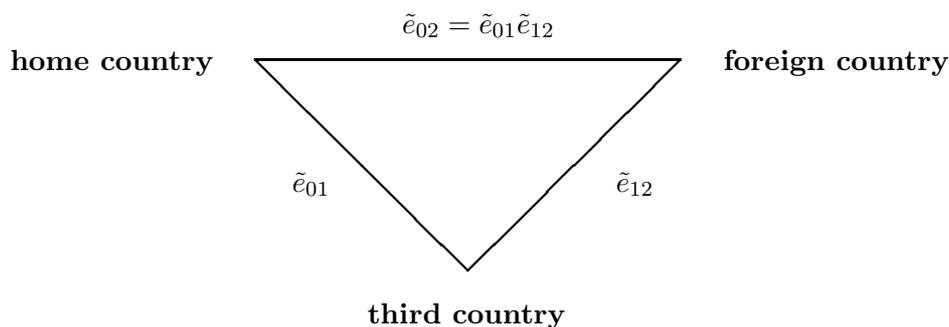
We assume that there are no hedging instruments directly related to the foreign currency. There is, however, a third country that has a currency futures market to which the MNF has access for cross-hedging purposes. We index the home country by 0, the third country by 1, and the foreign country by 2. Exchange rate uncertainty is modeled by three positive random variables, $\tilde{e}_{01}$, $\tilde{e}_{02}$, and $\tilde{e}_{12}$, where $\tilde{e}_{ij}$ is the spot exchange rate expressed in units of country i's currency per unit of country j's currency.[6] Based on $\tilde{e}_{01}$ and $\tilde{e}_{12}$, we can

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[5]Broll and Zilcha (1992) and Lien and Wong (2005) consider an alternative case that the MNF's home and foreign operations produce the same homogeneous good. None of the qualitative results are affected if we follow their approach except that we would have one-way trade between the home and foreign operations rather than two-way trade as in our model. See also Meng and Wong (2007) and Wong (2007b).

[6]An alternative way to model the exchange rate uncertainty is to apply the concept of information systems that are conditional cumulative distribution functions over a set of signals imperfectly correlated with $\tilde{e}_{ij}$ (see Eckwert and Zilcha, 2001, 2003; Drees and Eckwert, 2003; and Broll and Eckwert, 2006). The advantage

define a cross rate of the home currency against the foreign currency as $\tilde{e}_{01}\tilde{e}_{12}$. It follows immediately from the law of one price that $\tilde{e}_{02} = \tilde{e}_{01}\tilde{e}_{12}$. Figure 1 depicts such a triangular parity condition. Throughout the paper, we use a tilde ($\tilde{\ }$) to signify a random variable, whereas the realization of which does not have a tilde.

$$\tilde{e}_{02} = \tilde{e}_{01}\tilde{e}_{12}$$

home country $\qquad\qquad\qquad\qquad\qquad\qquad$ **foreign country**

$\tilde{e}_{01}$ $\qquad\qquad\qquad\qquad$ $\tilde{e}_{12}$

third country

Figure 1. Triangular parity condition

To cross-hedge its exchange rate risk exposure, the MNF can trade infinitely divisible currency futures contracts between the domestic and third currencies at date 0, each of which calls for delivery of e_{01}^{f} units of the domestic currency per unit of the third currency at date 1. Let h be the number of the currency futures contracts sold (purchased if negative) by the MNF at date 0. By convergence, the futures exchange rate at date 1 must be set equal to the spot exchange rate at that time, i.e., $\tilde{e}_{01}$. The gain (loss if negative) from the futures position, h, to the MNF is therefore given by $(e_{01}^{f} - \tilde{e}_{01})h$.

The MNF's random global profit at date 1, denominated in the domestic currency, is given by

$$\tilde{\pi} = r_x(x_h) + r_y(y_h) - c_x(x_h + x_f)$$

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of this more general and realistic approach is that one can study the value of information by comparing the information content of different information systems. Since the focus of this paper is not on the value of information, we adopt a simpler structure to save notation.

$$+\tilde{e}_{01}\tilde{e}_{12}[R_x(x_f) + R_y(y_f) - c_y(y_h + y_f)] + (e_{01}^f - \tilde{e}_{01})h, \tag{1}$$

where we have used the triangular parity condition that $\tilde{e}_{02} = \tilde{e}_{01}\tilde{e}_{12}$. The MNF possesses a von Neumann-Morgenstern utility function, $u(\pi)$, defined over its global domestic currency profit at date 1, π, with $u'(\pi) > 0$ and $u''(\pi) < 0$, indicating the presence of risk aversion.[7]

At date 0, the MNF chooses its levels of sales in the home and foreign markets, x_h, y_h, x_f, and y_f, and selects its futures position, h, so as to maximize the expected utility of its random global domestic currency profit at date 1:

$$\max_{x_h, y_h, x_f, y_f, h} \mathrm{E}[u(\tilde{\pi})], \tag{2}$$

where $\tilde{\pi}$ is defined in Eq. (1), and $\mathrm{E}(\cdot)$ is the expectation operator with respect to the joint cumulative distribution function of $\tilde{e}_{01}$ and $\tilde{e}_{12}$. The first-order conditions for program (2) with respect to x_h, x_f, y_h, y_f, and h are respectively given by

$$\mathrm{E}\{u'(\tilde{\pi}^*)[r_x'(x_h^*) - c_x'(x_h^* + x_f^*)]\} = 0, \tag{3}$$

$$\mathrm{E}\{u'(\tilde{\pi}^*)[\tilde{e}_{01}\tilde{e}_{12}R_x'(x_f^*) - c_x'(x_h^* + x_f^*)]\} = 0, \tag{4}$$

$$\mathrm{E}\{u'(\tilde{\pi}^*)[r_y'(y_h^*) - \tilde{e}_{01}\tilde{e}_{12}c_y'(y_h^* + y_f^*)]\} = 0, \tag{5}$$

$$\mathrm{E}\{u'(\tilde{\pi}^*)\tilde{e}_{01}\tilde{e}_{12}[R_y'(y_f^*) - c_y'(y_h^* + y_f^*)]\} = 0, \tag{6}$$

and

$$\mathrm{E}[u'(\tilde{\pi}^*)(e_{01}^f - \tilde{e}_{01})] = 0, \tag{7}$$

where an asterisk (*) signifies an optimal level.[8]

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[7]The risk-averse behavior of the MNF can be motivated by managerial risk aversion (Stulz, 1984), corporate taxes (Smith and Stulz, 1985), costs of financial distress (Smith and Stulz, 1985), and/or capital market imperfections (Stulz, 1990; and Froot, Scharfstein, and Stein, 1993). See Tufano (1996) for evidence that managerial risk aversion is a rationale for corporate risk management in the gold mining industry.

[8]The second-order conditions for program (2) are satisfied given risk aversion and the assumed properties of $r_x(x_h)$, $r_y(y_h)$, $R_x(x_f)$, $R_y(y_f)$, $c_x(x)$, and $c_y(y)$.

To focus on the MNF's hedging motive, vis-à-vis its speculative motive, we hereafter assume that the currency futures market is unbiased, i.e., we set $e_{01}^f = \mathrm{E}(\tilde{e}_{01})$.[9] Furthermore, to ease the exposition, we follow Broll, Wong, and Zilcha (1999) to assume that $\tilde{e}_{01}$ and $\tilde{e}_{12}$ are independent of each other. We can therefore write $\tilde{e}_{12} = \mathrm{E}(\tilde{e}_{12}) + \tilde{\varepsilon}$, where $\tilde{\varepsilon}$ is a zero-mean random variable independent of $\tilde{e}_{01}$.

3. The benchmark case of perfect hedging

In this section, we consider the benchmark case of perfect hedging in which the MNF has access to a currency futures market for the foreign currency. Specifically, each of the contracts that are traded in this currency futures market calls for delivery of e_{02}^f units of the domestic currency per unit of the foreign currency at date 1. We assume that the currency futures contracts between the domestic and foreign currencies are unbiased so that $e_{02}^f = \mathrm{E}(\tilde{e}_{02})$.

The MNF's random global profit at date 1, denominated in the domestic currency, is given by[10]

$$\tilde{\pi} = r_x(x_h) + r_y(y_h) - c_x(x_h + x_f)$$

$$+ \tilde{e}_{02}[R_x(x_f) + R_y(y_f) - c_y(y_h + y_f)] + [\mathrm{E}(\tilde{e}_{02}) - \tilde{e}_{02}]h, \tag{8}$$

where h is the number of the currency futures contracts between the domestic and foreign currencies sold (purchased if negative) by the MNF at date 0. The first-order conditions for the MNF's expected utility maximization problem are given by

$$\mathrm{E}\{u'(\tilde{\pi}^0)[r'_x(x_h^0) - c'_x(x_h^0 + x_f^0)]\} = 0, \tag{9}$$

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[9]If there are many risk-neutral speculators populated in the currency futures market, the unbiasedness of the futures exchange rate is an immediate consequence of no arbitrage opportunities.

[10]In the benchmark case of perfect hedging, even when the currency futures contracts between the domestic and third currencies are available for the MNF, we can easily verify that the MNF does not use them for hedging purposes.

$$\mathrm{E}\{u'(\tilde{\pi}^0)[\tilde{e}_{01}\tilde{e}_{12}R'_x(x_f^0) - c'_x(x_h^0 + x_f^0)]\} = 0, \tag{10}$$

$$\mathrm{E}\{u'(\tilde{\pi}^0)[r'_y(y_h^0) - \tilde{e}_{02}c'_y(y_h^0 + y_f^0)]\} = 0, \tag{11}$$

$$\mathrm{E}\{u'(\tilde{\pi}^0)\tilde{e}_{02}[R'_y(y_f^0) - c'_y(y_h^0 + y_f^0)]\} = 0, \tag{12}$$

and

$$\mathrm{E}\{u'(\tilde{\pi}^0)[\mathrm{E}(\tilde{e}_{02}) - \tilde{e}_{02}]\} = 0, \tag{13}$$

where $\tilde{\pi}^0$ is defined in Eq. (8), and a nought (^{0}) indicates an optimal level in the benchmark case of perfect hedging.

Solving Eqs. (9) to (13) yields our first proposition.

Proposition 1. *If the risk-averse MNF has access to the unbiased currency futures market for the foreign currency, then its optimal levels of sales in the home and foreign markets, x_h^0, x_f^0, y_h^0, and y_f^0, solve the following system of equations:*

$$r'_x(x_h^0) = c'_x(x_h^0 + x_f^0), \tag{14}$$

$$\mathrm{E}(\tilde{e}_{02})R'_x(x_f^0) = c'_x(x_h^0 + x_f^0), \tag{15}$$

$$r'_y(y_h^0) = \mathrm{E}(\tilde{e}_{02})c'_y(y_h^0 + y_f^0), \tag{16}$$

and

$$R'_y(y_f^0) = c'_y(y_h^0 + y_f^0), \tag{17}$$

and its optimal futures position, h^0, satisfies that $h^0 = R_x(x_f^0) + R_y(y_f^0) - c_y(y_h^0 + y_f^0)$.

Proof. Eqs. (14) and (17) follow from Eqs. (9) and (12), respectively. Substituting Eq. (13) into Eqs. (10) and (11) yields Eqs. (15) and (16), respectively. Finally, if

$h^0 = R_x(x_f^0) + R_y(y_f^0) - c_y(y_h^0 + y_f^0)$, it follows from Eq. (8) that $\pi^0 = r_x(x_h^0) + r_y(y_h^0) - c_x(x_h^0 + x_f^0) + \mathrm{E}(\tilde{e}_{02})[R_x(x_f^0) + R_y(y_f^0) - c_y(y_h^0 + y_f^0)]$, which is non-stochastic. Hence, $h^0 = R_x(x_f^0) + R_y(y_f^0) - c_y(y_h^0 + y_f^0)$ indeed solves Eq. (13) given the unbiasedness of the currency futures contracts between the domestic and foreign currencies. $\square$

The results of Proposition 1 are simply the celebrated separation and full-hedging theorems emanated from the literature on the MNF under exchange rate uncertainty (see, e.g., Broll, 1992; Adam-Müller, 1997; Broll and Wong, 1999, 2006; Broll, Wong, and Zilcha, 1999; and Wong, 2006). In the benchmark case of perfect hedging, the MNF's random global domestic currency profit at date 1 can be written as

$$\tilde{\pi} = r_x(x_h) + r_y(y_h) - c_x(x_h + x_f) + \mathrm{E}(\tilde{e}_{02})[R_x(x_f) + R_y(y_f) - c_y(y_h + y_f)]$$

$$+[\tilde{e}_{02} - \mathrm{E}(\tilde{e}_{02})][R_x(x_f) + R_y(y_f) - c_y(y_h + y_f) - h]. \tag{18}$$

Since the currency futures market for the foreign currency is unbiased, it offers actuarially fair "insurance" to the MNF. The risk-averse MNF as such optimally opts for full insurance by adopting a full-hedge, i.e., $h = R_x(x_f) + R_y(y_f) - c_y(y_h + y_f)$. Doing so completely eliminates the MNF's exposure to the exchange rate uncertainty, as is evident from Eq. (18). The MNF then optimally chooses it levels of sales in the home and foreign markets so as to maximize $r_x(x_h) + r_y(y_h) - c_x(x_h + x_f) + \mathrm{E}(\tilde{e}_{02})[R_x(x_f) + R_y(y_f) - c_y(y_h + y_f)]$, thereby yielding Eqs. (14) to (17). Eq. (14) states that the MNF equates the marginal cost of the good produced in the home operation to the marginal revenue of the good in the home market. Eqs. (14) and (15) imply that the marginal revenues of the good produced in the home operation, denominated in the domestic currency, are equalized in the home and foreign markets, where the spot exchange rate is locked in at the initial futures exchange rate, $\mathrm{E}(\tilde{e}_{02})$. Likewise, Eqs. (16) and (17) imply similar optimality conditions for the good produced in the foreign operation.

4. Optimal decisions under cross-hedging

We now resume our original case that the MNF has access to the currency futures market for the third currency only. We shall contrast the MNF's optimal decisions with those in the benchmark case of perfect hedging. Denote $E(\cdot|\cdot)$ and $\text{Cov}(\cdot, \cdot)$ as the conditional expectation operator and the covariance operator with respect to the joint cumulative distribution function of $\tilde{e}_{01}$ and $\tilde{e}_{12}$, respectively.

4.1 Optimal production and sales decisions

To examine the optimal production and sales decisions of the MNF in the case of cross-hedging, we solve Eqs. (3) to (7) to yield the following proposition

Proposition 2. *If the risk-averse MNF has access to the unbiased currency futures market for the third currency only, then its optimal levels of sales in the home and foreign markets, x_h^*, x_f^*, y_h^*, and y_f^*, solve the following system of equations:*

$$r_x'(x_h^*) = c_x'(x_h^* + x_f^*), \tag{19}$$

$$[E(\tilde{e}_{02}) - \theta]R_x'(x_f^*) = c_x'(x_h^* + x_f^*), \tag{20}$$

$$r_y'(y_h^*) = [E(\tilde{e}_{02}) - \theta]c_y'(y_h^* + y_f^*), \tag{21}$$

and

$$R_y'(y_f^*) = c_y'(y_h^* + y_f^*), \tag{22}$$

where $\theta = -\text{Cov}[u'(\tilde{\pi}^*), \tilde{e}_{01}\tilde{e}_{12}]/E[u'(\tilde{\pi}^*)] > 0$.

Proof. Eqs. (19) and (22) follow from Eqs. (3) and (6), respectively. Using the well-known property of the covariance operator, we can write Eqs. (4) and (5) as Eqs. (20) and (21), respectively.[11] Note that

$$\text{Cov}[u'(\tilde{\pi}^*), \tilde{e}_{01}\tilde{e}_{12}] = \text{E}[u'(\tilde{\pi}^*)\tilde{e}_{01}\tilde{e}_{12}] - \text{E}[u'(\tilde{\pi}^*)]\text{E}(\tilde{e}_{01}\tilde{e}_{12})$$

$$= \text{Cov}[u'(\tilde{\pi}^*)\tilde{e}_{01}, \tilde{e}_{12}] + \text{E}[u'(\tilde{\pi}^*)\tilde{e}_{01}]\text{E}(\tilde{e}_{12}) - \text{E}[u'(\tilde{\pi}^*)]\text{E}(\tilde{e}_{01})\text{E}(\tilde{e}_{12})$$

$$= \text{Cov}[u'(\tilde{\pi}^*)\tilde{e}_{01}, \tilde{e}_{12}]. \tag{23}$$

where the second equality follows from the independence of $\tilde{e}_{01}$ and $\tilde{e}_{12}$, and the third equality follows from Eq. (7). Note also that

$$\frac{\partial \text{E}[u'(\tilde{\pi}^*)\tilde{e}_{01}|e_{12}]}{\partial e_{12}} = \text{E}[u''(\tilde{\pi}^*)\tilde{e}_{01}^2|e_{12}][R_x(x_f^*) + R_y(y_f^*) - c_y(y_h^* + y_f^*)] < 0, \tag{24}$$

for all $e_{12} > 0$ given risk aversion, where we have used Eq. (1) and the independence of $\tilde{e}_{01}$ and $\tilde{e}_{12}$. Thus, Eqs. (23) and (24) imply that $\text{Cov}[u'(\tilde{\pi}^*), \tilde{e}_{01}\tilde{e}_{12}] < 0$. Hence, we conclude that $\theta > 0$. □

Comparing the set of optimality conditions in the case of cross-hedging, Eqs. (19) to (22), to that in the benchmark case of perfect hedging, Eqs. (14) to (17), yields the following proposition.

Proposition 3. *If the risk-averse MNF initially has access to the unbiased currency futures market for the third currency only, then introducing the unbiased currency futures market for the foreign currency to the MNF induces (i) greater sales of both goods in the home market, i.e., $x_h^* > x_h^0$ and $y_h^* > y_h^0$, (ii) lower sales of both goods in the foreign market, i.e., $x_f^* < x_f^0$ and $y_f^* < y_f^0$, (iii) lower output in the home operation, i.e., $x_h^* + x_f^* < x_h^0 + x_f^0$, and (iv) higher output in the foreign operation, i.e., $y_h^* + y_f^* > y_h^0 + y_f^0$.*

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[11]For any two random variables, $\tilde{x}$ and $\tilde{y}$, we have $\text{Cov}(\tilde{x}, \tilde{y}) = \text{E}(\tilde{x}\tilde{y}) - \text{E}(\tilde{x})\text{E}(\tilde{y})$.

Proof. Suppose first that $x_h^* \leq x_h^0$. Since $r_x''(x_h) < 0$, we have $r_x'(x_h^*) \geq r_x'(x_h^0)$. Eqs. (14) and (19) then imply that $c_x'(x_h^* + x_f^*) \geq c_x'(x_h^0 + x_f^0)$. Since $c_x''(x) > 0$ and $x_h^* \leq x_h^0$, we must have $x_f^* \geq x_f^0$. From Proposition 2, we know that $\theta > 0$. It then follows from $R_x''(x_f) < 0$ that $[\mathrm{E}(\tilde{e}_{02}) - \theta]R_x'(x_f^*) < \mathrm{E}(\tilde{e}_{02})R_x'(x_f^0)$. But this inequality together with Eqs. (15) and (20) would imply that $c_x'(x_h^* + x_f^*) < c_x'(x_h^0 + x_f^0)$, a contradiction. Hence, the supposition is not true and we must have $x_h^* > x_h^0$. It then follows from $r_x''(x_h) < 0$ that $r_x'(x_h^*) < r_x'(x_h^0)$. Eqs. (14) and (19) thus imply that $c_x'(x_h^* + x_f^*) < c_x'(x_h^0 + x_f^0)$. Since $c_x''(x) > 0$, we must have $x_h^* + x_f^* < x_h^0 + x_f^0$ and thereby $x_f^* < x_f^0$.

Now, suppose that $y_f^* \geq y_f^0$. Since $R_y''(y_f) < 0$, we have $R_y'(y_f^*) \leq R_y'(y_f^0)$. Eqs. (17) and (22) then imply that $c_y'(y_h^* + y_f^*) \leq c_y'(y_h^0 + y_f^0)$. Since $c_y''(y) > 0$ and $y_f^* \geq y_f^0$, we must have $y_h^* \leq y_h^0$. It then follows from $r_y''(y_h) < 0$ that $r_y'(y_h^*) \geq r_y'(y_h^0)$. But this inequality together with Eqs. (15) and (20) and $\theta > 0$ would imply that $c_y'(y_h^* + y_f^*) > c_y'(y_h^0 + y_f^0)$, a contradiction. Hence, the supposition is not true and we must have $y_f^* < y_f^0$. It then follows from $R_y''(y_f) < 0$ that $R_y'(y_f^*) > R_y'(y_f^0)$. Eqs. (17) and (22) thus imply that $c_y'(y_h^* + y_f^*) > c_y'(y_h^0 + y_f^0)$. Since $c_y''(y) > 0$, we must have $y_h^* + y_f^* > y_h^0 + y_f^0$ and thereby $y_h^* > y_h^0$. $\square$

The intuition of Proposition 3 is as follows. In the case of cross-hedging, the MNF cannot eliminate all the exchange rate risk and thus has to bear some residual risk. Such residual risk, however, can be controlled by varying the levels of sales in the home and foreign markets. Eq. (19) states that it remains optimal for the MNF to equate the marginal cost of the good produced in the home operation to the marginal revenue of the good in the home market. Eqs. (19) and (20), however, imply that the marginal revenue of the good produced in the home operation is strictly smaller in the home market than in the foreign market, where the latter is denominated in the domestic currency with the spot exchange rate equated to the expected level, $\mathrm{E}(\tilde{e}_{02})$. Since the sales in the foreign market are embedded with some exchange rate risk that cannot be eliminated due to imperfect hedging, the MNF has to demand a risk premium to compensate for its foreign sales. The

wedge between the two marginal revenues in the home and foreign markets is *de facto* the risk premium required by the MNF. Similar arguments apply to the good produced in the foreign operation. The MNF as such sells less (more) and produces more (less) in the foreign (home) country. These results are in line with the findings of Broll and Zilcha (1992), Lien and Wong (2005), Meng and Wong (2007), and Wong (2007b).

In the case of perfect hedging, the MNF optimally adopts a full-hedge that eliminates all the exchange rate risk. The MNF then chooses its sales and outputs so as to maximize its expected global domestic currency profit at date 1. When only cross-hedging is allowed, the MNF adjusts its sales and outputs according to Proposition 3 by selling less (more) and producing more (less) in the foreign (home) country. This must imply that the MNF's expected global domestic currency profit at date 1 is adversely affected in face of cross-hedging vis-à-vis perfect hedging. Given risk aversion, the MNF's expected utility must also be lowered. Thus, a lack of perfect hedging unambiguously makes the MNF strictly worse off regarding to both its expected profit and utility levels.

4.2 Optimal cross-hedging decisions

To examine the MNF's optimal cross-hedging decision, we use the covariance operator to write Eq. (7) with $e_{01}^f = E(\tilde{e}_{01})$ as

$$\text{Cov}[u'(\tilde{\pi}^*), \tilde{e}_{01}] = 0. \tag{25}$$

Note that

$$\frac{\partial E[u'(\tilde{\pi}^*)|e_{01}]}{\partial e_{01}} = E[u''(\tilde{\pi}^*)|e_{01}]\{E(\tilde{e}_{12})[R_x(x_f^*) + R_y(y_f^*) - c_y(y_h^* + y_f^*)] - h^*\}$$

$$+ E[u''(\tilde{\pi}^*)\tilde{\varepsilon}|e_{01}][R_x(x_f^*) + R_y(y_f^*) - c_y(y_h^* + y_f^*)], \tag{26}$$

where we have used Eq. (1) and $\tilde{e}_{12} = E(\tilde{e}_{12}) + \tilde{\varepsilon}$. Inspection of Eq. (26) reveals that the sign of $\text{Cov}[u'(\tilde{\pi}^*), \tilde{e}_{01}]$ depends on the sign of $u'''(\pi)$.

As convincingly argued by Kimball (1990, 1993), prudence, i.e., $u'''(\pi) \geq 0$, is a reasonable behavioral assumption for decision making under multiple sources of uncertainty. Prudence measures the propensity to prepare and forearm oneself under uncertainty, vis-à-vis risk aversion that is how much one dislikes uncertainty and would turn away from it if one could. As shown by Leland (1968), Drèze and Modigliani (1972), and Kimball (1990), prudence is both necessary and sufficient to induce precautionary saving. Furthermore, prudence is implied by decreasing absolute risk aversion, which is instrumental in yielding many intuitively appealing comparative statics under uncertainty (Gollier, 2001). We show in the following proposition that the MNF optimally opts for an under-hedge, i.e., $h^* < \mathrm{E}(\tilde{e}_{12})[R_x(x_f^*) + R_y(y_f^*) - c_y(y_h^* + y_f^*)]$, if its preferences exhibit not only risk aversion but also prudence.

Proposition 4. *If the risk-averse and prudent MNF has access to the unbiased currency futures market for the third currency only, then its optimal futures position, h^*, satisfies that $h^* < \mathrm{E}(\tilde{e}_{12})[R_x(x_f^*) + R_y(y_f^*) - c_y(y_h^* + y_f^*)]$.*

Proof. Given prudence, we have $\partial u''(\pi^*)/\partial\varepsilon = u'''(\pi^*)e_{01}[R_x(x_f^*) + R_y(y_f^*) - c_y(y_h^* + y_f^*)] \geq 0$. Since $\mathrm{E}(\tilde{\varepsilon}|e_{01}) = 0$, prudence implies that $\mathrm{E}[u''(\tilde{\pi}^*)\tilde{\varepsilon}|e_{01}] \geq 0$. If $h^* \geq \mathrm{E}(\tilde{e}_{12})[R_x(x_f^*) + R_y(y_f^*) - c_y(y_h^* + y_f^*)]$, it then follows from prudence and Eq. (26) that $\partial\mathrm{E}[u'(\tilde{\pi}^*)|e_{01}]/\partial e_{01} > 0$ for all $e_{01} > 0$ and thus $\mathrm{Cov}[u'(\tilde{\pi}^*), \tilde{e}_{01}] > 0$, a contradiction to Eq. (25). Hence, we conclude that $h^* < \mathrm{E}(\tilde{e}_{12})[R_x(x_f^*) + R_y(y_f^*) - c_y(y_h^* + y_f^*)]$. $\square$

The intuition of Proposition 4 is as follows. Suppose that the MNF adopts a full-hedge in that $h^* = \mathrm{E}(\tilde{e}_{12})[R_x(x_f^*) + R_y(y_f^*) - c_y(y_h^* + y_f^*)]$. It then follows from Eq. (1) with $e_{01}^f = \mathrm{E}(\tilde{e}_{01})$ and $\tilde{e}_{12} = \mathrm{E}(\tilde{e}_{12}) + \tilde{\varepsilon}$ that

$$\tilde{\pi}^* = r_x(x_h^*) + r_y(y_h^*) - c_x(x_h^* + x_f^*) + \mathrm{E}(\tilde{e}_{01})\mathrm{E}(\tilde{e}_{12})[R_x(x_f^*) + R_y(y_f^*) - c_y(y_h^* + y_f^*)]$$

$$+ \tilde{e}_{01}\tilde{\varepsilon}[R_x(x_f^*) + R_y(y_f^*) - c_y(y_h^* + y_f^*)]. \tag{27}$$

Such a full-hedge is not optimal for the MNF due to the residual risk, $\tilde{e}_{01}\tilde{\varepsilon}[R_x(x_f^*)+R_y(y_f^*)-c_y(y_h^*+y_f^*)]$, arising from a lack of perfect hedging, as is evident from Eq. (27). Given prudence, the MNF is more sensitive to low realizations of its random global domestic currency profit at date 1 than to high ones (Kimball, 1990, 1993). Note that the low realizations of its random global domestic currency profit at date 1 occur when the realized values of $\tilde{e}_{01}$ are large and those of $\tilde{\varepsilon}_1$ are negative. Inspection of Eq. (1) reveals that the MNF can avoid these realizations by shorting less of the currency futures contracts, i.e., $h^* < \mathrm{E}(\tilde{e}_{12})[R_x(x_f^*) + R_y(y_f^*) - c_y(y_h^* + y_f^*)]$, thereby rendering the optimality of an under-hedge.

Tufano (1996) documents that in the gold mining industry only 17% of firms shed 40% or more of their price risk. Bodnar, Hayt, and Marston (1998) also find that most non-financial firms do not use financial derivatives to completely hedge their risk exposures. The result of Proposition 4 is therefore consistent with such a normal practice of partial hedging.

5. Hedging role of options

In this section, we introduce currency put option contracts between the domestic and third currencies to the MNF as additional cross-hedging instruments.[12] Specifically, the MNF writes (purchases if negative) z units of the currency put option contracts with a single strike price, k, and an option premium, p, at date 0. At date 1, the gain (loss if negative) from the put option position, z, to the MNF is therefore given by $[p - \max(k - \tilde{e}_{01}, 0)]z$. To focus on the cross-hedging role of the currency put options, we assume that these contracts are fairly priced, i.e., $p = \mathrm{E}[\max(k - \tilde{e}_{01}, 0)]$.

The MNF's random global profit at date 1, denominated in the domestic currency, is

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[12]We do not consider currency call option contracts because they are redundant in that they can be readily replicated by combinations of currency futures and put option contracts (Sercu and Uppal, 1995).

given by

$$\tilde{\pi} = r_x(x_h) + r_y(y_h) - c_x(x_h + x_f) + \tilde{e}_{01}\tilde{e}_{12}[R_x(x_f) + R_y(y_f) - c_y(y_h + y_f)]$$

$$+[\mathrm{E}(\tilde{e}_{01}) - \tilde{e}_{01}]h + [p - \max(k - \tilde{e}_{01}, 0)]z. \tag{28}$$

At date 0, the MNF chooses its levels of sales in the home and foreign markets, x_h, y_h, x_f, and y_f, and selects its futures and put option positions, h and z, so as to maximize the expected utility of its random global domestic currency profit at date 1:

$$\max_{x_h, y_h, x_f, y_f, h, z} \mathrm{E}[u(\tilde{\pi})], \tag{29}$$

where $\tilde{\pi}$ is defined in Eq. (28).

The first-order conditions for program (29) with respect to x_h, x_f, y_h, y_f, h, and z are respectively given by

$$\mathrm{E}\{u'(\tilde{\pi}^{**})[r'_x(x_h^{**}) - c'_x(x_h^{**} + x_f^{**})]\} = 0, \tag{30}$$

$$\mathrm{E}\{u'(\tilde{\pi}^{**})[\tilde{e}_{01}\tilde{e}_{12}R'_x(x_f^{**}) - c'_x(x_h^{**} + x_f^{**})]\} = 0, \tag{31}$$

$$\mathrm{E}\{u'(\tilde{\pi}^{**})[r'_y(y_h^{**}) - \tilde{e}_{01}\tilde{e}_{12}c'_y(y_h^{**} + y_f^{**})]\} = 0, \tag{32}$$

$$\mathrm{E}\{u'(\tilde{\pi}^{**})\tilde{e}_{01}\tilde{e}_{12}[R'_y(y_f^{**}) - c'_y(y_h^{**} + y_f^{**})]\} = 0, \tag{33}$$

$$\mathrm{E}\{u'(\tilde{\pi}^{**})[\mathrm{E}(\tilde{e}_{01}) - \tilde{e}_{01}]\} = 0, \tag{34}$$

and

$$\mathrm{E}\{u'(\tilde{\pi}^{**})[p - \max(k - \tilde{e}_{01}, 0)]\} = 0, \tag{35}$$

where a double asterisk (**) signifies an optimal level.

We show in the following proposition that the MNF's optimal hedge position, (h^{**}, z^{**}), satisfies that $h^{**} < \mathrm{E}(\tilde{e}_{12})[R_x(x_f^{**}) + R_y(y_f^{**}) - c_y(y_h^{**} + y_f^{**})]$ and $z^{**} < 0$ if its preferences exhibit not only risk aversion but also prudence.

Proposition 5. *If the risk-averse and prudent MNF has access to the unbiased currency futures and options markets for the third currency only, then its optimal hedge position, (h^{**}, z^{**}), satisfies that $h^{**} < \mathrm{E}(\tilde{e}_{12})[R_x(x_f^{**}) + R_y(y_f^{**}) - c_y(y_h^{**} + y_f^{**})]$ and $z^{**} < 0$.*

Proof. Let $\tilde{e}_{01}$ be distributed according to a probability density function, $f(e_{01})$, over support $[\underline{e}_{01}, \overline{e}_{01}]$, where $0 \le \underline{e}_{01} < \overline{e}_{01} \le \infty$. To facilitate the exposition, we reformulate the MNF's ex-ante decision problem, program (29), as a two-stage optimization problem. In the first stage, we derive the optimal solution to program (29), $x_h(z)$, $y_h(z)$, $x_f(z)$, $y_f(z)$, and $h(z)$, as a function of z. In the second stage, we solve the MNF's optimal put option position, z^{**}, taking the first-stage solution as given. The complete solution to program (29) is thus given by z^{**}, $x_h^{**} = x_h(z^{**})$, $y_h^{**} = y_h(z^{**})$, $x_f^{**} = x_f(z^{**})$, $y_f^{**} = y_f(z^{**})$, and $h^{**} = h(z^{**})$.

Let $EU = \mathrm{E}[u(\tilde{\pi})]$, where $\tilde{\pi}$ is defined in Eq. (28) with $x_h = x_h(z)$, $y_h = y_h(z)$, $x_f = x_f(z)$, $y_f = y_f(z)$, and $h = h(z)$. Totally differentiating EU with respect to z, using the envelope theorem, and evaluating the resulting derivative at $z = 0$ yields

$$\left.\frac{\mathrm{d}EU}{\mathrm{d}z}\right|_{z=0} = \mathrm{E}\{u'(\tilde{\pi}^*)[p - \max(k - \tilde{e}_{01}, 0)]\}, \tag{36}$$

where $\tilde{\pi}^*$ is the optimal random domestic currency profit at date 1 when $z = 0$. Note that this case has been completely characterized in Section 4. If the right-hand side of Eq. (36) is negative (positive), it then follows from the strict concavity of EU that $z^{**} < (>) 0$.

Differentiating $\mathrm{E}[u'(\tilde{\pi}^*)|e_{01}]$ twice with respect to e_{01} yields

$$\frac{\partial^2 \mathrm{E}[u'(\tilde{\pi}^*)|e_{01}]}{\partial e_{01}^2} = \mathrm{E}\left\{u'''(\tilde{\pi}^*)\{\tilde{e}_{12}[R_x(x_f^*) + R_y(y_f^*) - c_y(y_h^* + y_f^*)] - h^*\}^2\Big|e_{01}\right\}, \tag{37}$$

which is non-negative given prudence. From Proposition 4, we know that h^* is less than

$\mathrm{E}(\tilde{e}_{12})[R_x(x_f^*) + R_y(y_f^*) - c_y(y_h^* + y_f^*)]$. Hence, Eqs. (26) and (37) implies that $\mathrm{E}[u'(\tilde{\pi}^*)|e_{01}]$ is U-shaped. Since $\mathrm{E}[u'(\tilde{\pi}^*)]$ is the expected value of $\mathrm{E}[u'(\tilde{\pi}^*)|e_{01}]$, which is U-shaped, there must be at least one and at most two distinct points in $(\underline{e}_{01}, \overline{e}_{01})$ at which $\mathrm{E}[u'(\tilde{\pi}^*)|e_{01}] = \mathrm{E}[u'(\tilde{\pi}^*)]$.

Suppose that $\mathrm{E}[u'(\tilde{\pi}^*)|e_{01}] = \mathrm{E}[u'(\tilde{\pi}^*)]$ at only one point, $\hat{e}_{01} \in (\underline{e}_{01}, \overline{e}_{01})$. We can write Eq. (25) as

$$\int_{\underline{e}_{01}}^{\overline{e}_{01}} \{\mathrm{E}[u'(\tilde{\pi}^*)|e_{01}] - \mathrm{E}[u'(\tilde{\pi}^*)]\}(\hat{e}_{01} - e_{01})f(e_{01})\,\mathrm{d}e_{01} = 0. \tag{38}$$

If $\mathrm{E}[u'(\tilde{\pi}^*)|\underline{e}_{01}] \leq \mathrm{E}[u'(\tilde{\pi}^*)]$, then $\mathrm{E}[u'(\tilde{\pi}^*)|e_{01}] < (>) \mathrm{E}[u'(\tilde{\pi}^*)]$ for all $e_{01} < (>)\ \hat{e}_{01}$ and thus the left-hand side of Eq. (38) is negative, a contradiction. On the other hand, if $\mathrm{E}[u'(\tilde{\pi}^*)|\underline{e}_{01}] > \mathrm{E}[u'(\tilde{\pi}^*)]$, then $\mathrm{E}[u'(\tilde{\pi}^*)|e_{01}] > (<) \mathrm{E}[u'(\tilde{\pi}^*)]$ for all $e_{01} < (>)\ \hat{e}_{01}$ and thus the left-hand side of Eq. (38) is positive, a contradiction. Hence, we conclude that $\mathrm{E}[u'(\tilde{\pi}^*)|e_{01}] = \mathrm{E}[u'(\tilde{\pi}^*)]$ at exactly two distinct points, e_{01}^* and e_{01}^{**}, where $\underline{e}_{01} < e_{01}^* < e_{01}^{**} < \overline{e}_{01}$.

Since $p = \mathrm{E}[\max{(k - \tilde{e}_{01}, 0)}]$, the right-hand side of Eq. (36) can be written as

$$F(k) = \int_{\underline{e}_{01}}^{k} \{\mathrm{E}[u'(\tilde{\pi}^*)] - \mathrm{E}[u'(\tilde{\pi}^*)|e_{01}]\}(k - e_{01})f(e_{01})\,\mathrm{d}e_{01}. \tag{39}$$

Differentiating Eq. (39) with respect to k and using Leibniz's rule yields

$$F'(k) = \int_{\underline{e}_{01}}^{k} \{\mathrm{E}[u'(\tilde{\pi}^*)] - \mathrm{E}[u'(\tilde{\pi}^*)|e_{01}]\}f(e_{01})\,\mathrm{d}e_{01}. \tag{40}$$

Differentiating Eq. (40) with respect to k and using Leibniz's rule yields

$$F''(k) = \{\mathrm{E}[u'(\tilde{\pi}^*)] - \mathrm{E}[u'(\tilde{\pi}^*)|k]\}f(k). \tag{41}$$

Using the fact that $\mathrm{E}[u'(\tilde{\pi}^*)|e_{01}]$ is U-shaped and $\mathrm{E}[u'(\tilde{\pi}^*)|e_{01}] = \mathrm{E}[u'(\tilde{\pi}^*)]$ at exactly two distinct points, e_{01}^* and e_{01}^{**}, it follows from Eq. (41) that $F''(k) > 0$ for all $k \in (e_{01}^*, e_{01}^{**})$ and $F''(k) \leq 0$ for all $k \notin (e_{01}^*, e_{01}^{**})$, where the equality holds only at $k = e_{01}^*$ and $k = e_{01}^{**}$. In words, $F(k)$ is strictly concave for all $k \in [\underline{e}_{01}, e_{01}^*) \bigcup (e_{01}^{**}, \overline{e}_{01}]$ and is strictly convex for

all $k \in (e^*_{01}, e^{**}_{01})$. It follows from Eq. (40) that $F'(\underline{e}_{01}) = F'(\overline{e}_{01}) = 0$. Hence, $F(k)$ attains two local maxima at $k = \underline{e}_{01}$ and $k = \overline{e}_{01}$. From Eq. (39), we have $F(\underline{e}_{01}) = 0$. Also, Eq. (39) implies that

$$F(\overline{e}_{01}) = \int_{\underline{e}_{01}}^{\overline{e}_{01}} \{E[u'(\tilde{\pi}^*)] - E[u'(\tilde{\pi}^*)|e_{01}]\}(\overline{e}_{01} - e_{01})f(e_{01}) \, de_{01}$$

$$= \text{Cov}[u'(\tilde{\pi}^*), \tilde{e}_{01}],$$

which vanishes by Eq. (25). In words, $F(k)$ has an inverted bell-shape bounded from above by zero at $k = \underline{e}_{01}$ and $k = \overline{e}_{01}$. Hence, $F(k) < 0$ for all $k \in (\underline{e}_{01}, \overline{e}_{01})$ and thus $z^{**} < 0$.

Using the covariance operator, we can write Eq. (34) as

$$\text{Cov}[u'(\tilde{\pi}^{**}), \tilde{e}_{01}] = 0. \tag{42}$$

Note that

$$\frac{\partial E[u'(\tilde{\pi}^{**})|e_{01}]}{\partial e_{01}} = E[u''(\tilde{\pi}^{**})|e_{01}]\Big\{E(\tilde{e}_{12})[R_x(x_f^{**}) + R_y(y_f^{**}) - c_y(y_h^{**} + y_f^{**})]$$

$$-h^{**} - \frac{\partial \max(k - e_{01}, 0)z^{**}}{\partial e_{01}}\Big\}$$

$$+E[u''(\tilde{\pi}^{**})\tilde{\varepsilon}|e_{01}][R_x(x_f^{**}) + R_y(y_f^{**}) - c_y(y_h^{**} + y_f^{**})], \tag{43}$$

where we have used Eq. (28). Given prudence, we have $\partial u''(\pi^{**})/\partial \varepsilon = u'''(\pi^{**})e_{01}[R_x(x_f^{**}) + R_y(y_f^{**}) - c_y(y_h^{**} + y_f^{**})] \geq 0$. Since $E(\tilde{\varepsilon}|e_{01}) = 0$, prudence implies that $E[u''(\tilde{\pi}^{**})\tilde{\varepsilon}|e_{01}] \geq 0$. If $h^{**} - z^{**} \geq E(\tilde{e}_{12})[R_x(x_f^{**}) + R_y(y_f^{**}) - c_y(y_h^{**} + y_f^{**})]$, it then follows from prudence and Eq. (43) that $\partial E[u'(\tilde{\pi}^{**})|e_{01}]/\partial e_{01} > 0$ for all $e_{01} > 0$ and thus $\text{Cov}[u'(\tilde{\pi}^{**}), \tilde{e}_{01}] > 0$, a contradiction to Eq. (42). Hence, we conclude that $h^{**} - z^{**} < E(\tilde{e}_{12})[R_x(x_f^{**}) + R_y(y_f^{**}) - c_y(y_h^{**} + y_f^{**})]$. Since $z^{**} < 0$, we must have $h^{**} < E(\tilde{e}_{12})[R_x(x_f^{**}) + R_y(y_f^{**}) - c_y(y_h^{**} + y_f^{**})]$. This completes our proof. $\square$

To see the intuition of Proposition 5, we rewrite Eq. (28) with $\tilde{e}_{12} = E(\tilde{e}_{12}) + \tilde{\varepsilon}$ as

$$\tilde{\pi}^{**} = E(\tilde{e}_{01})h^{**} + \tilde{e}_{01}\tilde{\varepsilon}[R_x(x_f^{**}) + R_y(y_f^{**}) - c_y(y_h^{**} + y_f^{**})]$$

$$+\tilde{e}_{01}\{\mathrm{E}(\tilde{e}_{12})[R_x(x_f^{**}) + R_y(y_f^{**}) - c_y(y_h^{**} + y_f^{**})] - h^{**}\}$$

$$+[p - \max{(k - \tilde{e}_{01}, 0)}]z^{**}. \tag{44}$$

Note that the second term on the right-hand side of Eq. (44), $\tilde{e}_{01}\tilde{\varepsilon}[R_x(x_f^{**}) + R_y(y_f^{**}) - c_y(y_h^{**} + y_f^{**})]$, is independent of $\tilde{e}_{01}$ and thus is not hedgeable by trading the currency futures and put options. Given prudence, the MNF is more sensitive to low realizations of its random global domestic currency profit at date 1 than to high ones (see Kimball, 1990, 1993). Note that the low realizations of its random global domestic currency profit at date 1 occur when the realized values of $\tilde{e}_{01}$ are large and those of $\tilde{\varepsilon}_1$ are negative. Inspection of Eq. (44) reveals that the MNF can avoid these realizations by shorting less of the currency futures contracts, i.e., $h^{**} < \mathrm{E}(\tilde{e}_{12})[R_x(x_f^{**}) + R_y(y_f^{**}) - c_y(y_h^{**} + y_f^{**})]$. Of course doing so would adversely affect the realizations of the MNF's random global domestic currency profit at date 1 when $\tilde{e}_{01}$ takes on small values. Hence, the MNF optimally opts for a long put option position, i.e., $z^{**} < 0$, to offset this adverse effect induced by under-hedging with the currency futures contracts.

Currency options are particularly useful for prudent MNFs in the case of cross-hedging because of their asymmetric payoff profiles, vis-à-vis the symmetric payoff profiles of currency futures. In the 1998 Wharton survey of financial risk management by US non-financial firms, Bodnar, Hayt, and Marston (1998) report that 68% of the 200 derivatives-using firms indicated that they had used some form of options within the past 12 months. In light of Proposition 5, a lack of perfect hedging is likely to account for the hedging demand for currency options by prudent MNFs (see also Chang and Wong, 2003).

6. Conclusion

In the post-Bretton Woods era, foreign exchange rates have been increasingly volatile, making exchange rate risk management a fact of financial life. This paper has examined the

impact of cross-hedging on the behavior of the risk-averse multinational firm (MNF) under multiple sources of exchange rate uncertainty along the lines of Broll and Zilcha (1992), Lien and Wong (2005), Meng and Wong (2007), and Wong (2007b). The MNF has operations domiciled in the home country and in a foreign country, each of which produces a single homogeneous good to be sold in the home and foreign markets. The MNF has access to a currency futures market between the domestic currency and a third currency only for cross-hedging purposes. Since a triangular parity condition holds among these three currencies, the MNF uses the available, yet incomplete, currency futures market to reduce its exchange rate risk exposure.

We have shown that the MNF optimally opts for an under-hedge should it be prudent in the sense of Kimball (1990, 1993). Furthermore, we have shown that the MNF demands a positive risk premium on its foreign sales, which creates a wedge between the marginal revenues in the home and foreign markets. In response to a lack of perfect hedging, the MNF optimally sells less (more) and produces more (less) in the foreign (home) country. The MNF as such receives a lower expected global domestic currency profit and attains a lower expected utility level than in the benchmark case of perfect hedging. These results are consistent with the findings of Broll and Zilcha (1992), Lien and Wong (2005), Meng and Wong (2007), and Wong (2007b).

When the set of hedging instruments made available to the MNF is expanded to include currency option contracts between the domestic and third currencies, we have shown that the MNF optimally opts for a long option position if it is prudent. We thus add to the burgeoning literature on the hedging role of options in that currency options are used by prudent MNFs in the case of cross-hedging (see also Chang and Wong, 2003).

In practice, MNFs, besides using cross-hedging, can employ international trade contracts and different types of contractual arrangements to reduce their exchange rate risk exposure. For example, they can accelerate (lead) or decelerate (lag) the timing of payments to reflect their expectations about future exchange rate movements. The effectiveness of the use of leads and lags may be suppressed because many governments impose limit on the

allowed range to prevent disruption of the flows of funds into or out of their countries.[13] Another common method of hiding exchange rate risk exposure is to invoice exports in the exporters' currency so as to pass the currency exposure on to the buyers. However, as shown in Donnenfeld and Zilcha (1991), invoicing exports in the importers' currency entails a precommitment to prices so that quantities to be delivered are invariant to realized spot exchange rates. Furthermore, Friberg (1998) shows that setting prices in the importers' currency is optimal when the elasticity of exchange rate pass-through is less than unity, which is the dominant empirical finding in the literature (Menon, 1995).[14]

Cross-hedging is important because it expands the opportunity set of hedging alternatives. Given the fact that currency derivative markets are not readily available in less developed countries and are just starting to develop in many of the newly industrializing countries of Latin America and Asia Pacific, it is clear that, for many MNFs exposed to currencies of these countries, cross-hedging will continue to be a major risk management technique for the reduction of their exchange rate risk exposure.[15]

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[13]For example, import leads are prohibited in Sweden. Italy, on the other hand, imposes a 180-day limit on export lags and a five-year limit on import lags to trade with non-OEDC countries.

[14]The extant empirical evidence shows that both the exporters' and the importers' currencies are used for invoicing. See, e.g., Grassman (1973, 1976), Magee (1974), Page (1977), van Nieuwkerk (1979), and Carse, Williamson, and Wood (1980).

[15]Eaker and Grant (1987) provide empirical evidence that cross-hedging is a useful technique to reduce exchange rate risk exposure.

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