Non-Stackelberg outcomes in Stackelberg duopoly experiments: A parsimonious explanation based on inequality aversion*

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Abstract:

In the Stackelberg duopoly experiments in Huck *et al.* (2001), the followers behave less timidly than predicted by conventional theory and the leaders act less aggressively than predicted. We provide a parsimonious explanation to these anomalies by simplifying the model of Fehr and Schmidt (1999) in two directions—there is no advantageous inequality aversion and all players with non-standard preferences have the same degree of disadvantageous inequality aversion. Maximum likelihood procedures show that the predictions of this model are consistent with the data in Huck *et al.* (2001), and that more than a third of the players have high degree of disadvantageous inequality aversion which is statistically different from zero.

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1 Introduction

In this paper we examine the implications of a particular theory of social preferences—inequality aversion (Fehr and Schmidt, 1999)—for Stackelberg duopoly experiments. In the experiments with Stackelberg quantity competition conducted by Huck *et al.* (2001), the outcome predicted by conventional theory (in which each player cares only about her material payoff) is rarely observed. Specifically, in one set of their random-matching experiments, in which anonymous subjects are randomly assigned the roles of leaders and followers in a Stackelberg game and the design ensures that each pair of players interact once, there are two major features inconsistent with conventional prediction. Conventional theory predicts that a follower, after observing the leader's output level, would react according to a downward-sloping best response function with a particular value of the slope. However, the followers behave according to this best response function in only 50.9% of the experiments. [1] On average, the followers behave *less timidly* than predicted, with the estimated response function less steep than the conventional one. Conventional theory also predicts that the Stackelberg leader would take the first-mover advantage and set the level of output at the profit-maximizing level. However, the leaders choose that value in less than 30% of the experiments. On average, the leaders behave *less aggressively* than predicted, with the average output level "between the subgame-perfect equilibrium and the symmetric Cournot equilibrium prediction" (Huck *et al.,* 2001, p. 757).

After presenting the experimental evidence, Huck *et al.* (2001) suggest that the inequality aversion model of Fehr and Schmidt (1999) may be helpful in explaining the subjects' behavior. They follow Fehr and Schmidt (1999) and postulate that some players have non-standard preferences in that they maximize a utility function consisting of material payoff and inequality aversion components. They argue that the Fehr and Schmidt (1999) model makes two predictions about the quantity-setting Stackelberg game. First, a follower chooses a quantity in the interval from the leader's choice to the best response against the leader's choice. Second, a leader chooses a quantity in the range from the profit-maximizing level to the collusive level.

[1] This number, which is not found in Huck *et al.* (2001), is calculated by us based on the data set provided by Steffen Huck. We are grateful to their kindness. Note that the major objective of Huck *et al.* (2001) is to use experimental evidence to compare the Cournot and Stackelberg quantity-setting duopoly markets; in particular, they examine a prediction (due to Daughety, 1990) that the total output level (and welfare) is higher in the Stackelberg market. They study the random-matching and fixed-matching treatments of both markets, and they mainly present the average level of output of the leaders and the followers. On the other hand, we focus on individual behavior of one set of their experiments—the Stackelberg games with random matching ("STACKRAND" in their notation).

In deriving the predictions about Stackelberg duopoly experiments, Huck *et al.*'s (2001) describe the entire range of equilibrium outcomes consistent with the Fehr and Schmidt (1999) model without specifying the distributions of players' inequality aversion parameters. While we share with their conjecture that inequality aversion may be important in explaining the behavior in these experiments, we think it is possible to obtain sharper predictions if one performs an analysis based on more specific distributions of the inequality aversion parameters. In this sense, this paper aims to complement their study. Besides providing sharper predictions of the quantity-setting Stackelberg game, an important contribution of this paper is that with the proposed model we are able to derive analytically the best response functions of different Stackelberg followers. These derivations lead naturally to a maximum-likelihood framework to test empirically the predictions of the model.

As indicated in the title of this paper, we aim to provide a parsimonious explanation based on inequality aversion. In scientific investigation, a parsimonious explanation has desirable properties since the effect of the factors under investigation is not confounded by other factors. Besides this general virtue, in the context of Stackelberg duopoly experiments, as the following analysis illustrates, the analysis with inequality aversion can be quite long and involved even for the relatively simple model considered in this paper, and will be more tedious for complicated models incorporating additional features. To overcome this problem, we choose to work with a simple model, and our approach is to select the assumptions supported by behavioral intuition and empirical evidence. Specifically, we model inequality aversion in the simplest way by (a) assuming that a proportion of the players has non-standard preferences, and that the non-standard preferences consist of disadvantageous inequality aversion but no advantageous inequality aversion; and (b) assuming that agents with such non-standard preferences will have the same disadvantageous inequality aversion parameter. This specification, while retaining the spirit of the Fehr and Schmidt (1999) model, simplifies the analysis substantially.

Our analysis of the simplified Fehr-Schmidt model reveals that even a small proportion of players with non-standard preferences is enough to cause non-Stackelberg outcomes, as well as that the Cournot outcome (in which the Stackelberg leader does not exploit the first-mover advantage at all) can arise rather easily. Compared with the conventional theory, this parsimonious model provides more accurate predictions of Stackelberg duopoly experiments. Based on the experimental results in Huck *et al.* (2001), we estimate that almost 40% of the Stackelberg followers have non-standard preferences and the disadvantageous inequality aversion parameter is very high and is significantly different from zero.

This paper is organized as follows. In Section 2 we summarize the behavior in Stackelberg duopoly experiments that conventional theory fails to explain and we propose a simple model. In Section 3 we derive the followers' best response functions. In Section 4 we study the leaders' behavior. In Sections 5 and 6 we compare the theoretical predictions with the laboratory evidence in Huck *et al.* (2001). We provide the concluding remarks in Section 7. We focus on economic intuitions in the main text, and leave the technical details in the Appendix.

2 The behavior to be explained, and the proposed model

In the experimental Stackelberg duopoly with quantity competition considered in this paper, a player—the Stackelberg leader L—commits her output level first. Knowing the choice of the leader, the other player—the Stackelberg follower F—selects her output. The profit (or material payoff) of player i is given by

$$z_i = z_i(q_i, q_j) = (\max\{d - q_i - q_j, 0\} - g) \times q_i, \tag{1}$$

where $i, j = L, F$ $(j \neq i)$, q_i is the output level of player i, and $d > g \geq 0$. One interpretation of (1) is that the price of a homogenous product either depends linearly on the sum of the two players' outputs or is zero (if the sum of the outputs is too high), and g is the constant marginal cost. Note that except for the different roles of the leader and follower, the two players are symmetric with the same material payoff function. In Huck *et al.* (2001), $d = 30$ and $g = 6$.

In our theoretical analysis, we assume that each player chooses the output level within a closed interval

$$q_i \in \left[\underline{q}, \overline{q}\right], \tag{2}$$

where $\underline{q} = 0$ and $\overline{q} = \frac{d-g}{2} \equiv \frac{h}{2}$. Note that within this interval, we have[2]

$$\max\{d - q_i - q_j, 0\} - g = h - q_i - q_j \geq 0. \tag{3}$$

We first summarize the conventional predictions regarding the above game. According to conventional theory, each player chooses her output to maximize her material payoff, given the behavior of her opponent. The subgame-perfect equilibrium predicted by conventional theory is as follows. Observing the level of output q_L selected by the leader, the Stackelberg follower chooses q_F to

[2] An implication of (A6a) in the Appendix and (11c) is that assuming $\overline{q} = \frac{h}{2}$ is not restrictive since output level higher than $\frac{h}{2}$ will not be chosen by the players. Allowing for $\overline{q} > \frac{h}{2}$ would not lead to different theoretical predictions but would make the analysis more tedious as we need to keep track of whether $h - q_L - q_F$ is positive or not at different output levels.

maximize $z_F = q_F (h - q_L - q_F)$. It is straightforward to show that the best response function of the follower of this quantity game is given by

$$q_F^S (q_L) = \frac{h}{2} - \frac{q_L}{2}. \tag{4}$$

We refer to the followers with "standard" preferences as type S followers. The best response function of type S followers is given in the upper panel of Figure 1. Anticipating that a follower will respond according to (4), the leader chooses q_L to maximize $z_L \left(q_L, q_F^S (q_L)\right)$. The optimal choice of the leader is given by $\frac{h}{2}$. Following Huck *et al.* (2001), we refer to this level as the "*profit-maximizing Stackelberg leader quantity.*"

While the conventional theory may be insightful in predicting behavior in some situations (such as R&D races in which history dictates who is the leader and follower), it is inconsistent with the experimental evidence in Huck *et al.* (2001). As mentioned in the Introduction, we focus on two anomalies of Stackelberg duopoly experiments with random matching in their paper. First, the estimated response function of the followers is much flatter than predicted. The estimated slope is significantly different from the predicted value of -0.5 given in (4). It is also significantly different from the predicted value of -0.49 for the discretized game in Huck *et al.* (2001); see their footnote 9. Second, the profit-maximizing Stackelberg leader quantity ($\frac{h}{2} = 12$) is only chosen in 27.3% of the 220 cases. The evidence is even worse if we focus on experienced players (by looking at the outcomes of Round 9, the second last round), who choose the profit-maximizing Stackelberg leader quantity in 13.6% of the trials. The average output level of the leaders is 10.19, almost 2 units below the profit-maximizing Stackelberg leader quantity. To summarize, according to the experimental evidence in Huck *et al.* (2001, Result 2), the Stackelberg leaders behave less aggressively than predicted, and the followers behave less timidly than predicted.

To explain these two anomalies, we consider an inequality aversion model (Fehr and Schmidt, 1999) in which each of the players cares about the absolute level of her payoff as well as how her payoff compared with her opponent's; see also Bolton (1991) and Bolton and Ockenfels (2000). The inequality aversion model, which has a relative payoff component, is particularly useful in explaining experimental evidence since the reference groups and outcomes in this context are reasonably clear. As mentioned in Fehr and Schmidt (1999, p. 822): "The subjects enter the laboratory as equals, they do not know anything about each other, and they are allocated to different roles in the experiment at random. Thus, it is natural to assume that the reference group is simply the set of subjects playing against each other and that the reference point, i.e., the equitable outcome, is given by the egalitarian outcome."

In Fehr and Schmidt (1999) and Huck *et al.* (2001, Section 4), agents with non-standard preferences have both aversion to disadvantageous and advantageous inequality components. Moreover, the distribution of individuals with different inequality aversion parameters can be quite general; see Table III of Fehr and Schmidt (1999) for an example. In order to provide a parsimonious explanation as well as to avoid unnecessarily tedious analysis, we focus on a simplified version of their model by making two modifications.

Following Fehr and Schmidt (1999), we assume that some players have the standard preferences as defined by the material payoff function in (1), but inequality aversion matters for the other players. We assume that proportion p of the players have non-standard preferences (called type N players) and the remaining proportion $1-p$ have standard preferences,[3] where

$$0 \leq p \leq 1. \tag{5}$$

Our first simplification of Fehr and Schmidt (1999) is that we assume type N players care about disadvantageous but not advantageous inequality aversion. Specifically, a type N player's utility level depends on her material payoff as well as a term capturing utility loss from disadvantageous inequality, as follows:

$$U_i = U_i\left(z_i, z_j\right) = z_i - \alpha_i \max\left\{z_j - z_i, 0\right\}, \tag{6}$$

where α_i is the disadvantageous inequality aversion parameter, and z_i is player i's material payoff given in (1). In general, both disadvantageous and advantageous inequality aversion may be important. However, according to the experimental evidence in Messick and Sentis (1985) and Loewenstein *et al.* (1989), the subjects exhibit an aversion to advantageous inequality which is significantly weaker than the aversion to disadvantageous inequality.[4] In light of these experimental evidence, we only consider aversion to disadvantageous inequality in our theoretical analysis.

Our second simplification of Fehr and Schmidt (1999) is that we specify a simple distribution of the disadvantageous inequality aversion parameters. We assume that for players with non-standard preferences, their inequality aversion parameter is the same. That is, $\alpha_i = a$ for all type N players, where

$$a > 0. \tag{7}$$

[3] Note that the conventional "self-interested" model is just a special case of the simplified Fehr-Schmidt model in that *all* players belong to the standard type. In Section 6 we test the null hypothesis that all players have the standard preferences against the alternative hypothesis of the simplified Fehr-Schmidt model.

[4] In the statistical analysis in Sub-section 6.2, we also allow for aversion to advantageous inequality. However, the estimated advantageous inequality aversion parameter is much smaller than the estimated disadvantageous inequality aversion parameter, and the null hypothesis that the advantageous inequality aversion parameter is 0 is not rejected.

To summarize, the utility function of a type N or type S player in the simplified Fehr-Schmidt model can be represented by (6), if we allow the disadvantageous inequality aversion parameter to be either positive or zero. Furthermore, the players' inequality aversion parameters in this model are distributed according to

$$\Pr(\alpha_i = a) = p; \ \ \Pr(\alpha_i = 0) = 1 - p. \tag{8}$$

3 Best response functions of the two types of followers

To examine the leaders' and followers' behavior in this model, our first step is to transform (6), in which player i's utility level depends on z_i and z_j (the players' material payoffs), into a form in which the utility level depends on q_i and q_j (the players' choice variables). Using (1) and (3), we have

$$\max\{z_j - z_i, 0\} = (h - q_i - q_j)\max\{q_j - q_i, 0\}. \tag{9}$$

Therefore, player i's utility can be expressed in terms of the players' choice variables as follows:

$$V_i(q_i, q_j) = U_i(z_i(q_i, q_j), z_j(q_j, q_i))$$

$$= (h - q_i - q_j)\,q_i - \alpha_i(h - q_i - q_j)\max\{q_j - q_i, 0\}. \tag{10}$$

In this section, we study the optimal response of a Stackelberg follower, after observing that the leader has chosen q_L. As the two types of followers have different inequality aversion parameters, their best response functions are different.

The best response function of type S followers is given by (4). On the other hand, the best response function of type N followers is defined by

$$q_F^N(q_L) = \arg\max_{q_F} V_F(q_F, q_L)$$

$$= \arg\max_{q_F}\left[(h - q_L - q_F)(q_F - \alpha_F \max\{q_L - q_F, 0\})\right]. \tag{11}$$

The analysis of the best response function of type N followers is more complicated since the utility loss due to disadvantageous inequality may or may not be relevant, depending on whether q_F is smaller than q_L or not. As shown in the Appendix, the form of the best response function of type N followers differs with respect to the choice of q_L. For convenience in subsequent analysis, we label the intervals $\left[\underline{q}, \frac{h}{3}\right]$, $\left[\frac{h}{3}, \widehat{q}\right]$, and $[\widehat{q}, \overline{q}]$ as Regions A, B, and C, respectively,[5] where

$$\widehat{q}=\left(\frac{1+a}{3+2a}\right)h. \tag{12}$$

If a leader chooses a low output level in Region A, it is shown in the Appendix that it will not be optimal for type N followers to choose q_F in $\left[\underline{q}, q_L\right)$. As a result, inequality aversion is irrelevant for them. Following similar analysis as in (4), if q_L is in Region A, then the best response function of type N followers is given by

$$q_F^N\left(q_L\right)=\frac{h}{2}-\frac{q_L}{2}. \tag{11a}$$

Note that $\frac{h}{3}$ (the highest level of q_L in Region A) is the intersection of (11a) and the 45-degree line ($q_F = q_L$). It is also the equilibrium output level of this game with symmetric utility functions, if both players care only about material payoff and they move simultaneously. This level is referred to as the *Cournot quantity.*

If a leader chooses a high output level outside Region A, it can be shown (in the Appendix) that it is not optimal for type N follower to choose q_F in the interval $(q_L, \overline{q}]$. As a result, inequality aversion becomes potentially relevant. In this case, if q_F is set according to (4), then (10) is not maximized because q_F would then be lower than q_L, causing a reduction of the follower's utility level through the second term in the right-hand side of (10). To reduce the effect due to disadvantageous inequality, the optimal choice of q_F is higher than that given in (4).

There is a qualitative difference in the behavior of type N followers when the leader chooses an output level outside Region A. In the Appendix, it is shown that if q_L is in Region B, the best response function of type N followers is upward sloping given by

$$q_F^N\left(q_L\right)=q_L. \tag{11b}$$

On the other hand, if q_L is in Region C, the best response function of type N followers is downward sloping given by

$$q_F^N\left(q_L\right)=\frac{h}{2}-\frac{q_L}{2\left(1+a\right)}. \tag{11c}$$

The intuition of the difference in (11b) and (11c) is as follows. There are two components in U_F according to (10) and a type N follower wants to maximize the first term (material payoff) but minimizes the second term (utility loss

[5] Note that $\frac{h}{3}$ belongs to both Regions A and B and $\widehat{q}$ belongs to both Regions B and C. The overlapping endpoints of the different regions do not cause problem since the functions we focus in subsequent analysis—$q_F^N\left(q_L\right)$ in (11a) to (11c), and $W_L\left(q_L\right)$ in (13a) to (13c)—are piecewise-continuous.

from disadvantageous inequality). To maximize the material payoff, the best response of the follower is given by (4). In Region B, whether the follower uses (4) or (11b), the difference in material payoff is relatively minor but the difference in the utility loss from inequality aversion is more important. Therefore, the choice of a type N follower that minimizes the utility loss due to disadvantageous inequality also maximizes (10), and the follower chooses (11b). In Region C, the reduction in material payoff if the follower uses (11b) is relatively large. As a result, minimizing the utility loss due to inequality aversion cannot compensate for the loss in the material payoff, and it is optimal for the follower to compromise between these two terms and choose according to (11c).

The best response function of a type N follower is represented in the lower panel of Figure 1. There are a number of interesting features. First, this function is continuous but is piecewise-linear instead of smooth throughout the interval $\left[\underline{q}, \overline{q}\right]$. Second, the best response function of a type N follower is upward sloping in Region B (with a slope of 1) but is downward sloping in Region C (and is less steep than Region A). As a result, the best response function of a type N follower lies above that of a type S follower in these two regions, meaning that a type N follower responds less timidly to the leader. Third, the best response function of type S followers is just a special case of that of type N followers, when parameter a approaches 0.

We summarize the results of this section in the following Proposition.

Proposition 1 The type N Stackelberg followers' best response function is a piecewise-linear one given by (11a) to (11c). In Region B ($\frac{h}{3} \leq q_L \leq \widehat{q}$) and Region C ($\widehat{q} \leq q_L \leq \overline{q}$), type N followers respond to q_L less timidly than type S followers.

How the Stackelberg leader chooses her output, anticipating the best response functions of these two types of followers, is given in the next section.

4 The Stackelberg leaders' behavior

In an experimental setting with both standard and non-standard types of players, it is natural to assume that an individual knows her type but not the type of her (anonymous) opponent. Moreover, instead of assuming that all players know in no uncertain way the value of parameters p and a, we make the less restrictive assumption that different players may have different beliefs of p and a.[6] While the Stackelberg leader does not know the identity of her opponent, she anticipates that the two types of followers will act according to (4) and (11), respectively. Conditional on the probability distribution (8), the Stackelberg leader chooses q_L to maximize the following "reduced-form" utility function:

$$\begin{aligned} W_L(q_L) &= E\left[V_L\left(q_L, q_F\left(q_L\right)\right)\right] \\ &= p \times U_L\left(q_L, q_F^N\left(q_L\right)\right) + (1-p) \times U_L\left(q_L, q_F^S\left(q_L\right)\right). \end{aligned} \tag{13}$$

Since the form of the best response function of type N followers differ in Regions A to C, we consider the behavior of the utility function $W_L(q_L)$ in each of the three regions and then combine the results.

For Region A ($\underline{q} \leq q_L \leq \frac{h}{3}$), it can be shown from (4), (11a), (13) and (A4) that

$$W_L(q_L) = W_L^A(q_L) = \frac{1}{4}(h - q_L)\left[(2 + 3\alpha_L) q_L - \alpha_L h\right]. \tag{13a}$$

From (13a), we have

Lemma 1. $W_L^A(q_L)$ in (13a) is increasing in $q_L \in \left[\underline{q}, \frac{h}{3}\right]$.[7]

For Region B ($\frac{h}{3} \leq q_L \leq \widehat{q}$), it can be shown from (4), (11b), (13) and (A2) that

$$W_L(q_L) = W_L^B(q_L) = q_L\left[p(h - 2q_L) + \frac{1}{2}(1-p)(h - q_L)\right]. \tag{13b}$$

For Region C ($\widehat{q} \leq q_L \leq \overline{q}$), one can show from (4), (11c), (13) and (A2) that

$$\begin{aligned} W_L(q_L) = W_L^C(q_L) &= p\left\{h - q_L - \left[\frac{h}{2} - \frac{q_L}{2(1+a)}\right]\right\} q_L + (1-p)\left[h - q_L - \left(\frac{h}{2} - \frac{q_L}{2}\right)\right] q_L \\ &= \left\{\frac{h}{2} - \frac{1}{2}\left[2 - E\left(\frac{1}{1+\alpha_F}\right)\right] q_L\right\} q_L, \end{aligned} \tag{13c}$$

where $E\left(\frac{1}{1+\alpha_F}\right)$ represents the leader's expectation of the term $\left(\frac{1}{1+\alpha_F}\right)$.[8]

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[6] Note that we would not be able to explain the diverse choice of q_L in this model with no aversion to advantageous inequality if we assume that all leaders have the same beliefs of p and a.

[7] The upward-sloping feature of function $W_L^A(q_L)$ in Region A can be traced to the presence of strategic substitutability and negative externality in the quantity game. The underlying reason is the same as that given in the paragraph after Proposition 3.

[8] Note that in (13c) α_F is interpreted as a random variable, which can take the value a (for type N followers) with probability p or 0 (for type S followers) with probability $1-p$.

It can be shown that each of the functions (13a) to (13c) is quadratic in q_L, and has a negative second derivative. As a result, the reduced-form utility function $W_L(q_L)$ is a continuous piecewise-quadratic function in $q_L \in \left[\underline{q}, \overline{q}\right]$ (but is non-differentiable at $q_L = \frac{h}{3}$ and $q_L = \widehat{q}$). From (13a) to (13c) and Lemma 1, we conclude that

Lemma 2. The disadvantageous inequality aversion parameter (α_L) of the Stackelberg leaders does not affect their optimal choices in the simplified Fehr-Schmidt model.

We now use the above results to predict the Stackelberg leaders' choices in this model.

4.1 The profit-maximizing Stackelberg leader quantity is not chosen

According to conventional theory with the assumptions that each player maximizes her material payoff, the Stackelberg leader is predicted to choose the profit-maximizing Stackelberg leader quantity $\frac{h}{2}$. On the other hand, the simplified Fehr-Schmidt model predicts that the profit-maximizing Stackelberg leader quantity will not be chosen if there are some type N players.[9] This is given in the following Proposition.

Proposition 2. The profit-maximizing Stackelberg leader quantity ($\frac{h}{2}$) will not be chosen in the simplified Fehr-Schmidt model, if p is strictly positive.

4.2 To exploit or not to exploit the first-mover advantage

A corollary of Lemma 1 is that the Stackelberg leader would not choose an output level in the interval $\left[\underline{q}, \frac{h}{3}\right)$, as any choice in this interval is dominated by $q_L = \frac{h}{3}$. According to Proposition 2, she would not choose the profit-maximizing Stackelberg leader quantity ($\frac{h}{2}$) if she expects that some followers are averse to disadvantageous inequality. Any choice in $\left[\frac{h}{3}, \frac{h}{2}\right)$ is still possible. Among these possible choices, the leader's payoff at the Cournot quantity ($q_L = \frac{h}{3}$) is the same as the follower's, but her payoff when $q_L \in \left(\frac{h}{3}, \frac{h}{2}\right)$ is higher than the follower's. Thus, one can interpret that the leader of the quantity-setting Stackelberg game exploits the first-mover advantage if $q_L \in \left(\frac{h}{3}, \frac{h}{2}\right)$, but does not exploit the first-mover advantage if $q_L = \frac{h}{3}$. In this sub-section, we

[9] Note that this result is derived on the assumption that the output choice is continuous. In footnote 14 we comment on this result if the choice variable is discretized, as in the experiments of Huck *et al.* (2001).

study the conditions under which the leader will (or will not) exploit the first-mover advantage. The question is also motivated by the empirical evidence in Huck *et al.* (2001). The conventional theory predicts that the leaders would never choose the Cournot quantity. However, in 17.3% of the cases, the leaders choose the Cournot quantity. For more experienced Stackelberg leaders (in Round 9), the Cournot quantity is observed in 22.7% of the trials.

In the following Proposition we obtain the necessary and sufficient conditions for the Stackelberg leader to choose the Cournot outcome and not to exploit the first-mover advantage. For convenience, we assume that if a leader is indifferent between choosing the Cournot outcome and exploiting the first-mover advantage, then she always chooses the Cournot outcome.

Proposition 3. (a) A necessary condition for the Stackelberg leader to choose the Cournot quantity ($\frac{h}{3}$) in the simplified Fehr-Schmidt model is

$$p \geq \frac{1}{3}. \tag{14}$$

(b) The necessary and sufficient conditions for the Stackelberg leader to choose the Cournot quantity in this model are (14) and

$$E\left(\frac{1}{1+\alpha_F}\right) \leq \frac{7}{8}. \tag{15}$$

Proposition 3(a) is closely related to the different behavior of the two types of followers in Region B—type S followers responds negatively to q_L according to (4), and type N followers responds positively to q_L according to (11b). A necessary condition for the Cournot quantity ($\frac{h}{3}$) to be chosen by the leader is that this value is not dominated by any other choice in Region B, or equivalently, $W_L(q_L)$ is downward sloping in Region B. According to (13), the leader's utility function $W_L(q_L)$ is a weighted average of $U_L\left(q_L, q_F^N(q_L)\right)$ and $U_L\left(q_L, q_F^S(q_L)\right)$. If all followers are of the standard type, it is easy to see that $U_L\left(q_L, q_F^S(q_L)\right)$ is upward sloping in Region B. This is because with only type S followers, strategic substitutability is present since the follower will respond negatively to q_L according to (4). We also know that negative externality is present in the relevant region of this quantity game. Using the terminology in Fudenberg and Tirole (1984), this is the "top dog" case (with negative externality and strategic substitutability).[10] From Fudenberg and Tirole (1984)

[10] Besides the various examples on industrial organization cited in Fudenberg and Tirole (1984), the importance of the sign of externality effect and the slope of the reaction function has also been discussed in many other situations, such as the relative payoffs of the Stackelberg leader and follower (Gal-Or, 1985; Dowrick, 1986) and macroeconomic coordination failures (Cooper and John, 1988).

and, especially, Lau (2001, Table 4), we know that the Stackelberg leader wants to increase q_L, or equivalently, $U_L\left(q_L, q_F^S(q_L)\right)$ is upward sloping in Region B. Because of negative externality, the leader wants to induce the follower to set a lower output; to achieve that, the leader wants to set a higher output since strategic substitutability is present. On the other hand, if all followers are of type N, they will respond positively to q_L. That is, strategic complementarity is present in Region B. This maps into the "puppy dog" case (with negative externality and strategic complementarity) of Fudenberg and Tirole's (1984) classification. According to Lau (2001, Table 4), the utility function $U_L\left(q_L, q_F^N(q_L)\right)$ is downward sloping in Region B (and thus the Stackelberg leader wants to decrease q_L). Combining these results, Proposition 3(a) has the intuitive appeal that the proportion of type N followers has to be higher than some threshold value $(\frac{1}{3})$ in order that the function $W_L(q_L)$ becomes downward sloping in Region B.

The intuition of Proposition 3(b) can be seen graphically. In Figure 2, we assume for convenience that there is a upper limit $\overline{a}$ for the aversion to disadvantageous inequality parameter. According to Proposition 3(b), the Cournot quantity is chosen when (14) and (15) are satisfied. Using $E\left(\frac{1}{1+\alpha_F}\right) = p\left(\frac{1}{1+a}\right) + (1-p)$, it is easy to see that (15) is equivalent to

$$a \geq \frac{1}{8p-1}. \tag{15a}$$

In terms of Figure 2, the leader would not exploit the first-mover advantage inside the area $GHIJ$, where the segment HI is represented by the downward-sloping line $a = \frac{1}{8p-1}$. According to Proposition 3(b), the leader would still exploit the first-mover advantage if the departure of standard preference in the direction of aversion to disadvantageous inequality is small, in the sense that either a or p is small. On the other hand, when at least a third of the players belongs to type N $(p \geq \frac{1}{3})$ and the inequality aversion parameter is reasonably far away from 0 $(a \geq \frac{1}{8p-1})$, the leader would choose the Cournot quantity and not exploit the first-mover advantage.

Is it likely that the Stackelberg leaders do not exploit the first-mover advantage according to the above model? Suppose that a Stackelberg leader thinks that half of the players are of type N, then she would not exploit the first-mover advantage if she expects $a \geq \frac{1}{3}$ according to (15a). If she expects that one-third of the players are of type N, then parameter a needs to be higher (since $a = \frac{1}{8p-1}$ is decreasing in p) for her not to exploit the first-mover advantage. Specifically, the critical value is $a = 0.6$. Relating parameter a to the more familiar ultimatum game (in which a player, the Proposer, makes a take-it-or-leave-it offer to the Responder of how to divide a fixed amount), $a \geq 0.6$ means that a type N Responder is willing to reject the Proposer's offer up to 27% of the fixed amount.[11] On the other hand, $a \geq \frac{1}{3}$ means that a type N player is willing to give up the offer if the Proposer offers to her an amount equal to or smaller than \$2 (out of a fixed amount of \$10). Numerous experimental evidence shows that a substantial number of Responders of the ultimatum game have actually turned down such offers (see Fehr and Schmidt (1999), Fehr and Gächter (2000), Camerer 2003, and the references therein).[12] An implication of these laboratory evidence is that many Stackelberg leaders may not exploit the first-mover advantage even if they anticipate that the extent of disadvantageous inequity aversion is quite moderate (e.g., $p = 0.5$ and $a = 0.5$), according to the simplified Fehr-Schmidt model.

5 Explaining the experimental results

We now examine whether the predictions of the simplified Fehr-Schmidt model provide a good explanation of the outcomes in the experimental Stackelberg duopoly (Huck *et al.,* 2001), and compare the predictions with those of the conventional theory. We also briefly comment on how these predictions compare with those of Huck *et al.* (2001, Section 4) in which the distributions of the inequality aversion parameters are not specified.

The conventional theory predicts that the followers would respond according to (4). In Huck *et al.* (2001), the followers respond according to the discretized version of (4) only in 50.9% of the experiments. The other half does not behave accordingly. On average, they behave less timidly. This is exactly what Proposition 1 predicts. According to it, type N followers would behave less timidly in Region B and C because they dislike disadvantageous inequality. A corollary of Proposition 1 is that in a population with both types of followers, the estimated linear best response function has a flatter slope, consistent with the evidence in Table 6 of Huck *et al.* (2001).[13]

Regarding the leaders' choice, conventional theory predict that they would choose the profit-maximizing Stackelberg leader quantity ($\frac{h}{2} = 12$) for the quantity-setting game. However, the leaders in Huck *et al.* (2001) make this

[11] Suppose that the fixed amount is \$10, and the Proposer offers \$$s$ ($s > 5$) to the Responder and keeps the remaining. A type N Responder would be indifferent about accepting or rejecting this offer when $0 = s - a\,[(10 - s) - s]$. Therefore, the critical value of s is given by $\frac{10a}{1+2a}$.

[12] For example, based on the outcomes across hundreds of trials, Fehr and Gächter (2000, p. 161) conclude that "Proposers who offer the Responder less than 30 percent of the available sum are rejected with a very high probability."

[13] In Sub-section 6.3 we will comment on the appropriateness of using the estimated slope and intercept of a linear regression model to test the conventional self-interested model, and we will suggest an alternative method.

choice only for 27.3% of the experiments. For more experienced players (in the second last round), the corresponding number is even lower at 13.6%. On the other hand, our parsimonious model suggests that the leader would behave less aggressively by choosing a lower output level, because of the fear that type N followers will behave less timidly than type S followers. Specifically, Lemma 1 and Proposition 2 imply that the leader may choose $q_L \in \left[\frac{h}{3}, \frac{h}{2}\right)$ depending on her expectation of the term $\left(\frac{1}{1+\alpha_F}\right)$. The prediction that q_L is between 8 to 11 fits 50.9% of the experiments in Huck *et al.* (2001). If we consider the evidence of more experienced players, the prediction fits 72.8% of the cases.[14] Judging from the leaders' choice, the simplified Fehr-Schmidt model also performs better than the conventional theory.

Huck *et al.* (2001, Section 4) make two predictions of the Stackelberg duopoly based on the Fehr and Schmidt (1999) model without specifying a distribution about the inequality aversion parameters. First, they predict that the followers' choice are between q_L and the (conventional) best response to q_L. Based on a model with only disadvantageous inequality aversion and only one value of a for players with non-standard preferences, we obtain a sharper prediction of the best response function for the followers with non-standard preferences. As given in (11a) to (11c), it is first downward sloping, then upward sloping and then downward sloping again.

Huck *et al.* (2001) also predict that the Stackelberg leader's choice is between the collusive quantity ($\frac{h}{4}$ in this quantity-setting game) and the profit-maximizing Stackelberg leader quantity. There are two differences between their predictions and ours. The first difference is that the profit-maximizing Stackelberg leader quantity is not chosen in our model (Proposition 2) if the choice variable is continuous. The second difference is that in this model with no aversion to advantageous inequality, the Stackelberg leader would not choose $q_L < \frac{h}{3}$. Thus, our prediction is inconsistent with 10% of the trials in Huck *et al.* (2001) in which q_L is between 3 to 7.[15] This is the price, arguably a relatively small one, we have to pay for this simplified Fehr-Schmidt model.

[14] The predictions are even better if we allow for the fact that Huck *et al.* (2001) use a discretized version of the quantity-setting game. While the leaders would not choose $q_L = \frac{h}{2}$ according to our model if the output variable is continuous, it is possible that a leader with an optimal $q_L \in (11, 12)$ would choose $q_L = 12$ in the discretized game if $W_L(11) < W_L(12)$. For the discretized game, the prediction that q_L is between 8 to 12 fits 78.2% (and 86.4% for more experienced players) of the experiments in Huck *et al.* (2001).

[15] The predictions of the leaders' behavior are also inconsistent with 11.8% of the trials in which q_L is between 13 and 15. However, the behavior in these trials cannot be explained even if aversion to advantageous inequality is present.

6 Formal tests of the simplified Fehr-Schmidt model

As a major motivating factor of introducing the simplified Fehr-Schmidt model is to explain the anomalies in Huck *et al.*'s (2001) experiments, in Section 5 we focus on the predictions of the model regarding these anomalies. As such, other testable implications of the model have not been explored there. A logical next step is to see whether we can rigorously test the predictions of this social-preferences model, just like the implications of the conventional self-interested model have been examined in the literature. We show in this section that our theoretical analysis can be used to develop formal statistical testing procedures to examine the validity of these predictions.

In this section we focus on the behavior of the experimental Stackelberg followers. As shown in the analysis of Section 4, the predicted action of the Stackelberg leader depends on her expectation of the follower's type. Thus, any test about these predictions is a joint test of the underlying theoretical structure and the assumed expectations formation mechanism. On the other hand, the predictions about the follower's choice in Section 3 is purely based on the assumed behavioral model, since the follower has already observed the leader's action. In the following analysis we focus on the clean predictions about the followers' behavior, which is based on their preferences only and is not confounded by other auxiliary assumptions.

6.1 The likelihood function of a sample of Stackelberg followers' choices

To obtain the maximum likelihood estimates of the parameters of the simplified Fehr-Schmidt model, we first derive the likelihood function of observing the followers' choices conditional on the leaders' actions in a sample of n independent experimental trials as described in Section 2. In the statistical model we interpret that the theoretical predictions in Section 3 form the systematic part of the followers' behavior, and we introduce a random error term to capture other unspecified influences on their behavior.

In the following analysis we use x and y to stand for q_L and q_F, respectively. Let x_i and y_i represent, respectively, the leader and follower's choices for the i-th $(i = 1, ..., n)$ observation. If the follower in the i-th trial has standard preferences, then

$$y_i = q_F^S(x_i) + \varepsilon_i, \tag{16}$$

where the best response function $q_F^S(.)$ is given by (4) and the random error ε_i is assumed to be independently and identically distributed according to a normal distribution $N(0, \sigma^2)$. On the other hand, if the follower in the i-th trial has non-standard preferences, then

$$y_i = q_F^N(x_i) + \varepsilon_i, \tag{17}$$

where the best response function $q_F^N(.)$ is given by (11a) to (11c), depending on the value of the leader's choice x_i.

According to our model, the population of Stackelberg followers consists of proportion $1-p$ of players with standard preferences, and proportion p of players with non-standard preferences. Thus, from an econometrician's perspective, the probability density of observing y_i (conditional on x_i and parameters a, p and σ) is given by

$$(1-p) \times f_S(y_i|\, x_i; \sigma) + p \times f_N(y_i|\, x_i; a, \sigma), \tag{18}$$

where $f_S(y_i|\, x_i; \sigma)$ is the probability density of observing y_i when the follower has standard preferences, and $f_N(y_i|\, x_i; a, \sigma)$ is the probability density of observing y_i when the follower has non-standard preferences.

From now on, simply write $f_S(y_i|\, x_i; \sigma)$ as $f_S(y_i)$ and $f_N(y_i|\, x_i; a, \sigma)$ as $f_N(y_i)$. From (16), (17), (4) and (11a) to (11c), it is easy to obtain [16]

$$f_S(y_i) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[\frac{-\left(y_i - \frac{h}{2} + \frac{x_i}{2}\right)^2}{2\sigma^2}\right], \tag{19}$$

and

$$f_N(y_i) = f_S(y_i)^{1-D_B(x_i)-D_C(x_i)} \times f_B(y_i)^{D_B(x_i)} \times f_C(y_i)^{D_C(x_i)}, \tag{20}$$

where

$$D_B(x_i) = \begin{cases} 1 \text{ if } \frac{h}{3} < x_i \leq \left(\frac{1+a}{3+2a}\right) h \\ 0 \quad\quad \text{otherwise} \end{cases}, \tag{21}$$

$$D_C(x_i) = \begin{cases} 1 \text{ if } \left(\frac{1+a}{3+2a}\right) h < x_i \\ 0 \quad\quad \text{otherwise} \end{cases}, \tag{22}$$

$$f_B(y_i) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[\frac{-(y_i - x_i)^2}{2\sigma^2}\right], \tag{23}$$

[16] In the theoretical analysis, we restrict $\overline{q} = \frac{h}{2}$ to simplify the derivation; see footnote 2. However, we allow $x > \frac{h}{2}$ in the statistical analysis (with $h = 24$), since 11.8% of the leaders in Huck *et al.* (2001) choose $x > \frac{h}{2} = 12$. The conclusions of our statistical analysis remain unchanged whether we include the trials with $x > 12$ or not.

and

$$f_C(y_i) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[\frac{-\left(y_i - \frac{h}{2} + \frac{x_i}{2(1+a)}\right)^2}{2\sigma^2}\right]. \tag{24}$$

Therefore, the (log) likelihood function of observing a sample of n independent Stackelberg experiments is given by:

$$\ln L\left(a, p, \sigma; (x_1, y_1), ..., (x_n, y_n)\right)$$

$$= \sum_{i=1}^{n} \ln \left\{(1-p) f_S(y_i) + p\left[f_S(y_i)^{1-D_B(x_i)-D_C(x_i)} f_B(y_i)^{D_B(x_i)} f_C(y_i)^{D_C(x_i)}\right]\right\}. \tag{25}$$

The maximum likelihood estimates of the parameters of the simplified Fehr-Schmidt model are obtained by maximizing the log likelihood function (25). Even though (25) is highly nonlinear, the estimates can easily be obtained by standard optimization procedures. Applying it to the random-matching Stackelberg experiments of Huck *et al.* (2001), the maximum likelihood estimates are given in Table 1.[17] In particular, the estimates of p and a are 0.386 and 5.231, respectively.

Before performing further statistical analysis regarding these estimation results, we examine in the next sub-section whether the simplified Fehr-Schmidt model is adequate or not when compared with a more general specification including aversion to advantageous inequality.

6.2 Is aversion to advantageous inequality unimportant?

In the analysis of earlier sections, we omit aversion to advantageous inequality. To see whether this omission is appropriate or not, we now compare our

[17] There are two caveats in applying the likelihood function (25) to the Huck *et al.* (2001) data. First, this likelihood function is derived for the material payoff functions given by (1), but the payoff structure in Huck *et al.*'s (2001) experiments has been modified slightly to assure uniqueness of the equilibrium; see p. 753 of their paper. Second, the derivation of (25) is based on n (=220) independent trials. However, the random-matching Stackelberg experiment data set in Huck *et al.* (2001) consists of 22 pairs of players, each with 10 trials. If we had the coding of individual subjects, it might be better to assume that the same follower would use either (16) or (17) in all 10 trials. Unfortunately, such detailed information for this data set has been discarded (private communication). The best we can do in this situation is to use (25), and this can still be justified if we make the (perhaps more controversial) assumption that a follower may use (17) with probability p and (16) with probability $1-p$ in different trials.

parsimonious model with a more general model with aversion to advantageous inequality as well. In this case, the follower's payoff function is given by:

$$U_F = U_F(z_F, z_L) = z_F - \alpha_F \max\{z_L - z_F, 0\} - \beta_F \max\{z_F - z_L, 0\}, \quad (26)$$

where β_F is the advantageous inequality aversion parameter. Moreover, instead of (8), we assume

$$\Pr(\alpha_F = a \ \& \ \beta_F = b) = p; \ \Pr(\alpha_F = \beta_F = 0) = 1 - p \quad (27)$$

where p and a are given in (5) and (7) respectively, and

$$b \geq 0. \quad (28)$$

Following similar steps as before, it can be shown that the log likelihood function corresponding to this model is given by

$$\ln L = \sum_{i=1}^{n} \ln\left\{(1-p) f_S(y_i) + p\left[f_A(y_i)^{1-D_B(x_i)-D_C(x_i)} f_B(y_i)^{D_B(x_i)} f_C(y_i)^{D_C(x_i)}\right]\right\}, \quad (29)$$

where

$$f_A(y_i) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[\frac{-\left(y_i - \frac{h}{2} + \frac{x_i}{2(1-b)}\right)^2}{2\sigma^2}\right], \quad (30)$$

$$D_B(x_i) = \begin{cases} 1 \text{ if } \left(\frac{1-b}{3-2b}\right) h < x_i \leq \left(\frac{1+a}{3+2a}\right) h \\ 0 \qquad \text{otherwise} \end{cases}, \quad (31)$$

and $f_S(y_i)$, $f_B(y_i)$, $f_C(y_i)$ and $D_C(x_i)$ are the same as before.

The maximum likelihood estimates of this model are given in Table 1. It is noted that the estimated value of b is very small (0.156) and the other estimates are very close to those of our parsimonious model. To test formally the hypothesis that aversion to advantageous inequality is absent (i.e., $b = 0$), we construct the usual test statistic $t = \frac{\widehat{b}}{se(\widehat{b})}$, where $\widehat{b}$ is the maximum likelihood estimate of b, and $se(\widehat{b})$ is the standard error. Under the null hypothesis $b = 0$, this test statistic has a standard normal distribution asymptotically. Based on the estimated results in Table 1, it is easy to conclude that the null hypothesis $b = 0$ is not rejected at, say, a 5% significance level.[18]

[18] It is easy to conclude from Table 1 that the same conclusion is obtained if we use the likelihood ratio test.

6.3 Testing the self-interested model against the simplified Fehr-Schmidt model

Huck *et al.* (2001) examine the conventional self-interested model by using a linear regression model of the followers' choices. The estimated intercept and slope (in their Table 6) are significantly different from the conventional predictions. In their specification the null hypothesis of the self-interested model is tested against an alternative model with a linear best response function different from the conventional prediction. However, it is not clear what the behavioral model corresponding to this alternative hypothesis is.

Guided by the theoretical analysis in Section 3, we believe that a better way to examine the validity of the self-interested model is to use the simplified Fehr-Schmidt model (instead of an unspecified model with a linear best response function) to nest the self-interested model. To test the null hypothesis of the self-interested model against the alternative hypothesis of the simplified Fehr-Schmidt model, we obtain the likelihood function under the null hypothesis as

$$\ln L\left(\sigma;\left(x_1,y_1\right),...,\left(x_n,y_n\right)\right)=\sum_{i=1}^{n}\ln\left[f_S\left(y_i\right)\right], \tag{32}$$

where $f_S\left(y_i\right)$ is given by (19). The maximum likelihood estimates of this model for the Huck *et al.* (2001) data is given in Table 1.

To test the null hypothesis of the self-interested model against the simplified Fehr-Schmidt model, we construct the likelihood ratio test statistic

$$LR=2\left(\ln L_u-\ln L_r\right), \tag{33}$$

where the unrestricted log likelihood $\ln L_u$ is given by (25) and the restricted log likelihood $\ln L_r$ is given by (32). Under the null hypothesis, the test statistic of (33) has a χ^2-distribution of 2 degrees of freedom. Based on the estimation results in Table 1, $LR = 184.88$. Therefore, the null hypothesis is strongly rejected.[19]

To conclude, the self-interested model is decisively rejected against the simplified Fehr-Schmidt model, based on the Huck *et al.* (2001) experimental results. Moreover, the estimates of both p and a are significantly different from zero. According to Table 1, there is a representative group of followers (38.6%) with non-standard preferences, and the estimated disadvantageous inequality aversion parameter is a high value of 5.231.

[19] We obtain the same conclusion based on the asymptotically equivalent Wald test. An advantage of using the likelihood ratio test is that the standard deviation parameter σ under the null hypothesis can be estimated. According to Table 1, the estimate of σ under the self-interested model (2.591) is much larger than that under the simplified Fehr-Schmidt model (1.164).

7 Conclusion

In the random-matching Stackelberg duopoly experiments in Huck *et al.* (2001), the players do not behave as predicted by the conventional theory. Specifically, the followers act less timidly than predicted and the leaders behave less aggressively than predicted. This behavioral pattern raises the questions of why the conventional theory fails, and how to provide a better explanation.

In the Stackelberg duopoly experiments (as well as many other game-theoretic situations), testing the predictions of conventional theory is really testing the joint hypothesis of strategic behavior and the self-interested model. In the current case, the structure of the game is very simple (especially for the followers) that it is reasonable to maintain the assumption that the players are intelligent enough to understand the game and behave strategically. As a result, it is natural to seek for explanations based on non-standard preferences.

While there are well-known models of social preferences in the literature (such as Andreoni (1990) and the models cited above), there are also some critics arguing against this approach, as they are worried about altering the unobservable utility function would allow one to explain any phenomenon. In his book on behavioral economics, Camerer (2003, p. 101) defends the study of social preferences convincingly: "The goal is *not* to explain every different finding by adjusting the utility function just so; the goal is to find parsimonious utility functions, supported by psychological intuition, that are general enough to explain many phenomena in one fell swoop, and also make new predictions." See also footnote 10 of Bolton and Ockenfels (2000).

In this paper, we follow this idea and develop a parsimonious model of social preferences, which nests the standard preferences as a special case, to explain experimental Stackelberg duopoly. We simplify the well-received model of Fehr and Schmidt (1999) in two directions—there is only one (representative) group of non-standard players and there is no advantageous inequality aversion. With these simplifications, we are able to obtain sharper predictions about the behavior of the leaders and followers, and to explain the major features of Huck *et al.* (2001) better than conventional theory.

Another contribution of this paper is that the derivation of the best response functions of different types of Stackelberg followers leads naturally to a maximum likelihood framework to examine the implications of our parsimonious model. We then perform more structural estimations of this model with the Stackelberg duopoly experimental data in Huck *et al.* (2001).[20] These formal

[20] Cox *et al.* (2005) also perform structural estimations with this data set, but their specification not only allows for inequality aversion, but also a concern for the opponent's intentions (Rabin, 1993).

statistical analyses reveal that the implications of the simplified Fehr-Schmidt model are consistent with the data, and that while the behavior of the majority of the players is consistent with conventional theory, a significant proportion (close to 40%) of the players are averse to disadvantageous inequality.

8 Appendix [21]

Derivation of the best response function of type N followers. As the utility function of a type N player is different for $q_F < q_L$ and $q_F \geq q_L$, we analyze these two cases separately (for each of the three regions). Note that when $q_F < q_L$ (and therefore $z_F < z_L$), we have

$$U_F = z_F - \alpha_F (z_L - z_F) = (h - q_L - q_F)\left[(1+\alpha_F) q_F - \alpha_F q_L\right] \tag{A1}$$

and

$$U_L = z_L = (h - q_L - q_F)\, q_L. \tag{A2}$$

On the other hand, when $q_F \geq q_L$, we have

$$U_F = z_F = (h - q_L - q_F)\, q_F \tag{A3}$$

and

$$U_L = z_L - \alpha_L (z_F - z_L) = (h - q_L - q_F)\left[(1+\alpha_L) q_L - \alpha_L q_F\right]. \tag{A4}$$

First, consider Region A when the leader's output q_L lies in $\left[\underline{q}, \frac{h}{3}\right]$. Using (A1), we show that for any q_L in this interval, U_F is increasing in q_F for $q_F \in [0, q_L]$. Using (A3), we show that U_F is first increasing in q_F for $q_F \in \left[q_L, \frac{h}{2} - \frac{q_L}{2}\right]$ and then decreasing in q_F for $q_F \in \left[\frac{h}{2} - \frac{q_L}{2}, \overline{q}\right]$. Thus, the best response of a type N follower to $q_L \in \left[\underline{q}, \frac{h}{3}\right]$ is given by (11a).

Second, consider Region C. Using (A1), we show that for any q_L in $[\widehat{q}, \overline{q}]$, U_F is first increasing in q_F for $q_F \in \left[\underline{q}, \frac{h}{2} - \frac{q_L}{2(1+a)}\right]$ and then decreasing in q_F for $q_F \in \left[\frac{h}{2} - \frac{q_L}{2(1+a)}, q_L\right]$. Using (A3), we show that U_F is decreasing in q_F for $q_F \in [q_L, \widehat{q}]$. Thus, the best response of a type N follower to $q_L \in [\widehat{q}, \overline{q}]$ is given by (11c). Note that $\widehat{q}$ in (12) is given by the intersection of (11c) and the 45-degree line ($q_F = q_L$); see Figure 1 also.

Third, consider Region B. Using (A1), we show that for any q_L in $\left[\frac{h}{3}, \widehat{q}\right]$, U_F is increasing in q_F for $q_F \in \left[\underline{q}, q_L\right]$. Using (A3), we show that U_F is decreasing in

[21] In the following analysis we make use of well-known properties of quadratic functions.

q_F for $q_F \in [q_L, \overline{q}]$. Thus, the best response of a type N follower to $q_L \in \left[\frac{h}{3}, \widehat{q}\right]$ is given by (11b). ■

Proof of Lemma 1. It is easy to see that the quadratic function $W_L^A(q_L)$ in (13a) has a negative second derivative. Define q_L^A, which is not necessarily restricted to Region A, as the value of q_L such that the first derivative of $W_L^A(q_L)$ is zero. Simple algebra show that

$$q_L^A = \frac{(1+2\alpha_L)}{(2+3\alpha_L)}h. \tag{A5}$$

As a result, we have

$$q_L^A - \frac{h}{3} = \frac{(1+3\alpha_L)}{3(2+3\alpha_L)}h > 0. \tag{A5a}$$

Since $\frac{d^2 W_L^A(q_L)}{dq_L^2} < 0$ and $q_L^A > \frac{h}{3}$, the function $W_L^A(q_L)$ must be monotonic increasing in Region A (from $\underline{q}$ to $\frac{h}{3}$). This proves Lemma 1. ■

Proof of Proposition 2. Define q_L^C, which is not necessarily restricted to Region C, as the value of q_L such that the first derivative of $W_L^C(q_L)$ in (13c) is zero. It can be shown that

$$q_L^C = \frac{1}{2\left[2 - E\left(\frac{1}{1+\alpha_F}\right)\right]}h. \tag{A6}$$

A necessary condition for the leader to choose the profit-maximizing Stackelberg leader quantity ($\frac{h}{2}$) is that $q_L^C \geq \frac{h}{2}$. However, when p is strictly positive, it can be shown that

$$\frac{h}{2} - q_L^C = \frac{h}{2} - \frac{1}{2\left[2 - \left(\frac{1+a-ap}{1+a}\right)\right]}h = \left(\frac{ap}{1+a+ap}\right)\frac{h}{2} > 0. \tag{A6a}$$

Since $q_L^C < \frac{h}{2}$, we conclude that $q_L = \frac{h}{2}$ must be dominated by some other values of q_L in Region C, regardless of whether q_L^C is in Region C or not. This proves Proposition 2. ■

Proof of Proposition 3. A necessary condition for $q_L = \frac{h}{3}$ (Cournot quantity) to be chosen is that $W_L^B(q_L)$ in (13b) is downward sloping in Region B. Define q_L^B, which is not necessarily restricted to Region B, as the value of q_L such that the first derivative of $W_L^B(q_L)$ is zero. It is easy to show that

$$q_L^B = \frac{(1+p)}{2(1+3p)}h. \tag{A7}$$

Therefore,

$$\frac{h}{3} - q_L^B = \frac{(3p-1)}{6(1+3p)}h. \tag{A7a}$$

Since $W_L^B(q_L)$ is downward-sloping in Region B if and only if q_L^B is less than or equal to $\frac{h}{3}$, it is easy to conclude from (A7a) that this is satisfied when $p \geq \frac{1}{3}$. This proves Proposition 3(a).

When (14) is satisfied, Lemma 1 and Proposition 3(a) imply that $q_L = \frac{h}{3}$ is not dominated by any other choices in Regions A and B. Using either (13a) or (13b), it can be shown that the leader's utility level at $q_L = \frac{h}{3}$ is

$$W_L^A\left(\frac{h}{3}\right) = W_L^B\left(\frac{h}{3}\right) = \frac{1}{9}h^2. \tag{A8}$$

The leader would choose the Cournot quantity $\left(\frac{h}{3}\right)$ when this choice is not dominated by any other choice in Region C. If $q_L^C \leq \widehat{q}$ (and thus $W_L^C(q_L)$ is downward sloping in Region C), this is automatically satisfied.[22] If q_L^C is in Region C, then the leader's maximum utility level in this interval is given by

$$W_L^C\left(q_L^C\right) = \frac{1}{8\left[2 - E\left(\frac{1}{1+\alpha_F}\right)\right]}h^2. \tag{A9}$$

Thus, it is necessary and sufficient for $q_L = \frac{h}{3}$ to be chosen when the value of (A8) is higher than or equal to that of (A9). Simple algebra shows that this is equivalent to (15). This proves Proposition 3(b). ■

References

[1] Andreoni, J. (1990), "Impure altruism and donations to public goods: A theory of warm-glow giving," *Economic Journal* 100, 464-471.

[2] Bolton, G. E. (1991), "A comparative model of bargaining: Theory and evidence," *American Economic Review* 81, 1096-1136.

[3] Bolton, G. E. and A. Ockenfels (2000), "ERC: A theory of equity, reciprocity, and competition," *American Economic Review* 90, 166-193.

[4] Camerer, C. (2003), *Behavioral Game Theory: Experiments in Strategic Interaction.* Princeton: Princeton University Press.

[22] It can be shown that $q_L^C \leq \widehat{q}$ is equivalent to $a \geq \frac{1}{2p}$. Thus, this inequality and (14) are sufficient for the Cournot quantity to be chosen by the Stackelberg leader. Note that the curve $a = \frac{1}{2p}$, which is equivalent to $q_L^C = \widehat{q}$, is drawn in Figure 2.

[5] Cooper, R. and A. John (1988), "Coordinating coordination failures in Keynesian models," *Quarterly Journal of Economics* 103, 441-463.

[6] Cox, J. C., D. Friedman and S. Gjerstad (2005), "A tractable model of reciprocity and fairness," working paper.

[7] Daughety, A. F. (1990), "Beneficial concentration," *American Economic Review* 80, 1231-1237.

[8] Dowrick, S. (1986), "Von Stackelberg and Cournot duopoly: Choosing roles," *Rand Journal of Economics* 17, 251-260.

[9] Fehr, E. and S. Gächter (2000), "Fairness and retaliation: The economics of reciprocity," *Journal of Economic Perspectives* 14(3), 159-81.

[10] Fehr, E. and K. Schmidt (1999), "A theory of fairness, competition, and cooperation," *Quarterly Journal of Economics*, 114, 817-868.

[11] Fudenberg, D. and J. Tirole (1984), "The fat-cat effect, the puppy-dog Ploy, and the lean and hungry look," *American Economic Review Papers and Proceedings* 74, 361-366.

[12] Gal-Or, E. (1985), "First mover and second mover advantages," *International Economic Review* 26, 649-653.

[13] Huck, S., W. Müller and H.-T. Normann (2001), "Stackelberg beats Cournot–On collusion and efficiency in experimental markets," *Economic Journal* 111, 749-765.

[14] Lau, S.-H. P. (2001), "Aggregate pattern of time-dependent adjustment rules, II: Strategic complementarity and endogenous nonsynchronization," *Journal of Economic Theory* 98, 199-231.

[15] Loewenstein, G. F., L. Thompson and M. H. Bazerman (1989), "Social utility and decision making in interpersonal contexts," *Journal of Personality and Social Psychology* 57, 426-441.

[16] Messick, D. M. and K. P. Sentis (1985), "Estimating social and nonsocial utility functions from ordinal data," *European Journal of Social Psychology* 15, 389-399.

[17] Rabin, M. (1993), "Incorporating fairness into game theory and economics," *American Economic Review* 83, 1281-1302.

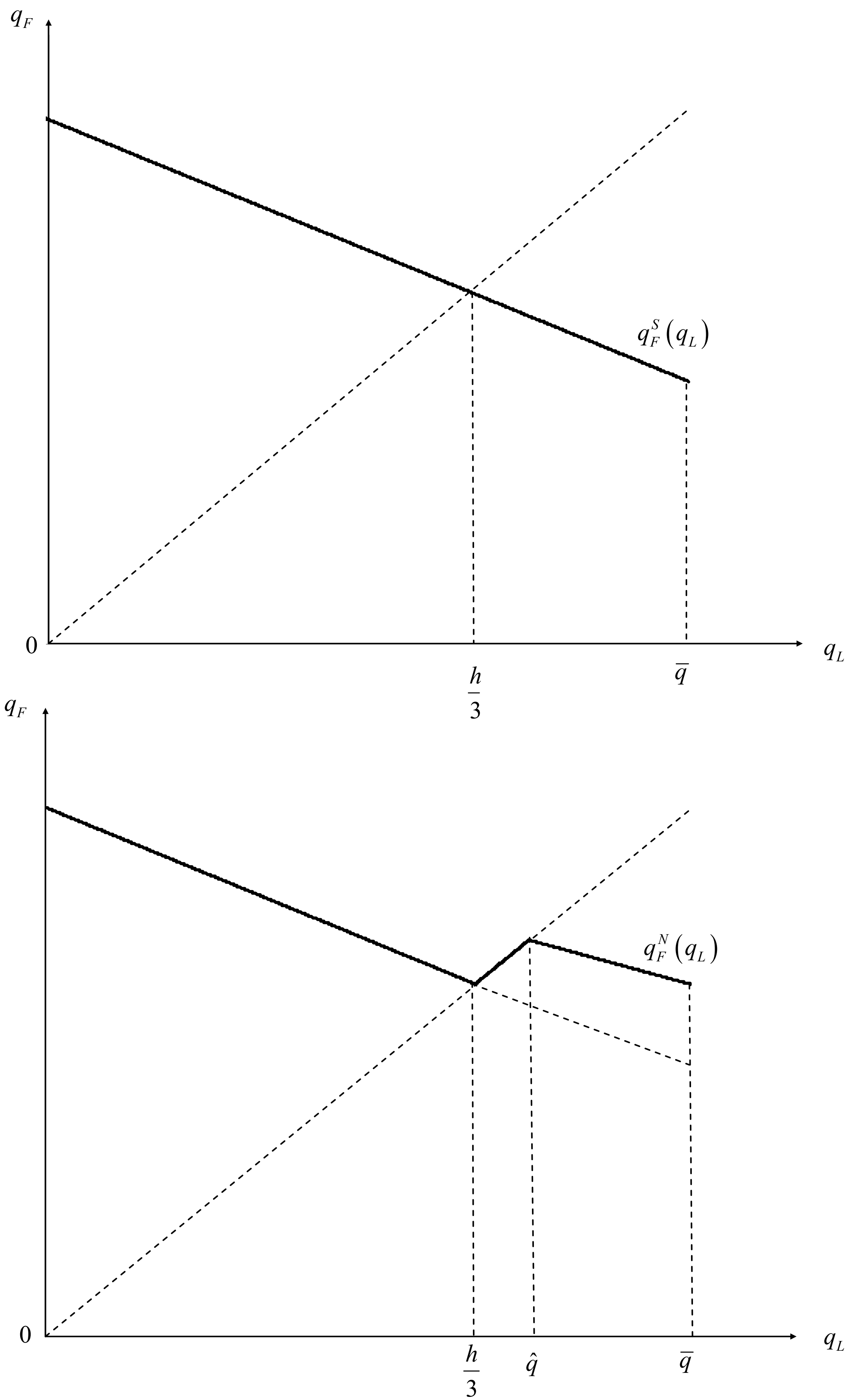

Figure 1: Best Response Functions of Type S and Type N Followers

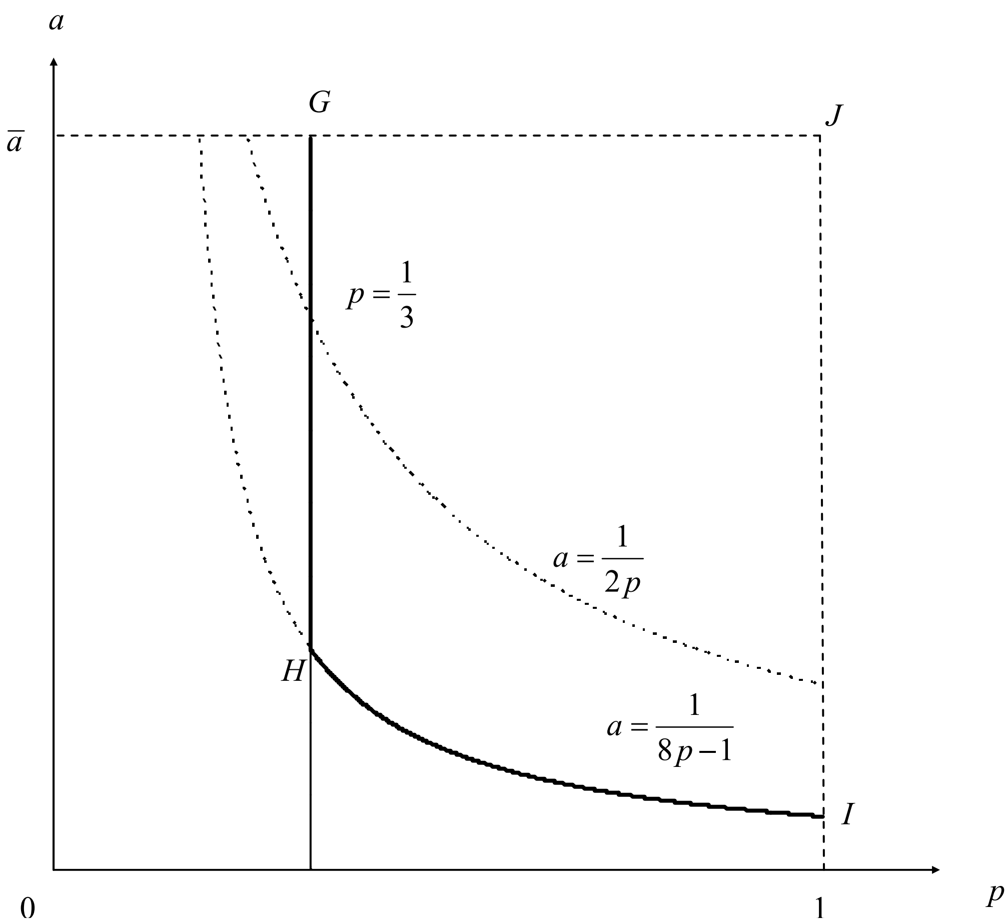

Figure 2: Exploiting or Not Exploiting the First-Mover Advantage

	Simplified Fehr-Schmidt Model	Model with Aversion to Advantageous Inequality	Self-Interested Model
p	0.386 (0.0427)	0.388 (0.0427)	-
a	5.231 (1.323)	5.231 (1.316)	-
b	-	0.156 (0.114)	-
σ	1.164 (0.0619)	1.158 (0.0617)	2.591 (0.124)
ln L	-429.14	-428.52	-521.58

Note: For each coefficient, the number not in parentheses is the maximum likelihood estimate and the number in parentheses is the standard error. The estimation is based on 220 trials of the random-matching Stackelberg experiments in Huck *et al.* (2001); see footnote 17 also.

Table 1: Estimation Results