Optimal Export and Hedging Decisions When Forward Markets Are Incomplete *

Kit Pong WONG †
University of Hong Kong

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This paper examines the behavior of the competitive firm that exports to two foreign countries under multiple sources of exchange rate uncertainty. There is a forward market between the home currency and one foreign country's currency, but there are no hedging instruments directly related to the other foreign country's currency. We show that the separation theorem holds when the firm optimally exports to the foreign country with the currency forward market. The full-hedging theorem holds when the firm either exports exclusively to the foreign country with the currency forward market or when the relevant spot exchange rates are independent. In the case that the relevant spot exchange rates are positively (negatively) correlated in the sense of regression dependence, the firm optimally opts for a short (long) forward position for cross-hedging purposes.

JEL classification: D81; F23; F31

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† *Correspondence to:* Kit Pong Wong, School of Economics and Finance, University of Hong Kong, Pokfulam Road, Hong Kong. Tel.: (852) 2859-1044, fax: (852) 2548-1152, e-mail: kpwong@econ.hku.hk (K. P. Wong).

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Abstract

This paper examines the behavior of the competitive firm that exports to two foreign countries under multiple sources of exchange rate uncertainty. There is a forward market between the home currency and one foreign country's currency, but there are no hedging instruments directly related to the other foreign country's currency. We show that the separation theorem holds when the firm optimally exports to the foreign country with the currency forward market. The full-hedging theorem holds when the firm either exports exclusively to the foreign country with the currency forward market or when the relevant spot exchange rates are independent. In the case that the relevant spot exchange rates are positively (negatively) correlated in the sense of regression dependence, the firm optimally opts for a short (long) forward position for cross-hedging purposes.

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1. Introduction

The effect of currency forward markets on the behavior of the competitive exporting firm under exchange rate uncertainty has been extensively studied in the literature (see, e.g., Adam-Müller, 1997; Broll, 1992; Broll, Wong, and Zilcha, 1999; Broll and Zilcha, 1992; Katz and Paroush, 1979; Wong, 2002, 2003). Two notable results emanate. First, the separation theorem states that the firm's production decision depends neither on its risk attitude nor on the underlying exchange rate uncertainty when the firm has access to a currency forward market for hedging purposes. Second, the full-hedging theorem states that the firm should completely eliminate its exchange rate risk exposure by adopting a full-hedge if the currency forward market is unbiased.

In many less developed countries (LDCs), capital markets are embryonic and foreign exchange markets are heavily controlled. Currency forward markets as such are seldom readily available in LDCs.[1] Also, in many of the newly industrializing countries of Latin America and Asia Pacific, currency forward markets are just starting to develop in a rather slow pace (Eiteman, Stonehill, and Moffett, 2004). Exporting firms that expose to currencies of these countries thus have to avail themselves of forward contracts on related currencies to cross-hedge their exchange rate risk exposure (Anderson and Danthine, 1981; Broll, 1997; Broll and Eckwert, 1996; Broll and Wong, 1999; Chang and Wong, 2003; Eaker and Grant, 1987).

The purpose of this paper is to examine the optimal export and hedging decisions of the competitive exporting firm in the case of incomplete currency forward markets. To this end, we assume that the firm exports to two foreign countries under multiple sources of exchange rate uncertainty. Forward markets are incomplete in the sense that there are only forward contracts between the home and one foreign country's currencies. Within this framework, we show that the separation theorem holds when the firm optimally exports to the foreign country with the currency forward market. The full-hedging theorem holds when the firm either exports exclusively to the foreign country with the currency forward market or when the relevant spot exchange rates are independent. In the case that the relevant spot exchange rates are positively (negatively) correlated in the sense of regression dependence (Lehmann, 1966), the firm optimally opts for a short (long) forward position for cross-hedging purposes.[2]

Since the seminal paper of Anderson and Danthine (1981), cross-hedging has become an important strand of the hedging literature. Adam-Müller (1997), Briys, Crouhy, and Schlesinger (1993), Broll and Eckwert (1996), and Broll, Wong, and Zilcha (1999) look at the case where the multiple sources of uncertainty are independent. Broll and Wong (1999), Chang and Wong (2003), and Wong (2003) allow linear dependence structure to ac-

[1]Even if some LDCs have currency forward contracts, these contracts are deemed to be forward-cover insurance schemes that are not governed by market forces (Jacque, 1996).

[2]Regression dependence describes the form of non-linear dependence structure of two random variables in general and how they behave together when their realizations are simultaneously small or large in particular. Shea (1979) shows that positive (negative) regression dependence of two random variables implies positive (negative) correlation between them, albeit not the converse.

commodate correlated sources of uncertainty. The contribution of this paper is to introduce general non-linear dependence structure that enables intuitive prescriptions of cross-hedging strategies for exporting firms.

Broll, Wong, and Zilcha (1999) study the behavior of the competitive firm that exports to two foreign countries within the same framework as ours. Unlike us, Broll, Wong, and Zilcha (1999) impose two additional assumptions that the relevant spot exchange rates are independent and the expected marginal revenues from exports to the two foreign countries are the same. In this regard, their model is a special case of ours. Broll, Wong, and Zilcha (1999) show that the firm exports only to the foreign country with the currency forward market. Indeed, we show that this is true irrespective of whether the relevant spot exchange rates are independent or not. Furthermore, we fully characterize the firm's optimal production and export decisions without putting any restrictions on the underlying exchange rate uncertainty and on the marginal revenues from exports to the two foreign countries, thereby making our results completely general.

The rest of the paper is organized as follows. Section 2 describes our model of the competitive exporting firm under multiple sources of exchange rate uncertainty. As a benchmark, Section 3 examines the firm's optimal export and hedging decisions when currency forward markets are complete. Section 4 derives the firm's optimal export and cross-hedging decisions in response to forward market incompleteness. The final section concludes. All proofs of propositions are given in the appendix.

2. The model

Consider the competitive exporting firm that produces a single output, x, according to a deterministic cost function, $c(x)$, in the home country, where $c(0) \geq 0$, $c'(x) > 0$, and $c''(x) > 0$. The firm sells its entire output, x, to two foreign countries, indexed by $i = 1$ and 2. Let x_i be the amount of exports sold in country i ($i = 1$ and 2), where $x_i \geq 0$ and

$x_1 + x_2 = x$. The per-unit selling price in country i, denoted by p_i, is exogenously fixed and denominated in country i's currency.[3] Due to the segmentation of the two foreign markets, arbitrage transactions are either impossible or unprofitable, thereby hindering the law of one price.[4]

We model the multiple sources of exchange rate uncertainty by two random spot exchange rates, $\tilde{e}_1$ and $\tilde{e}_2$, where $\tilde{e}_i$ is expressed in units of the home currency per unit of country i's currency ($i = 1$ and 2).[5] We use $\mathrm{E}(\cdot)$ and $\mathrm{Cov}(\cdot, \cdot)$ to denote the expectation operator and the covariance operator with respect to the joint probability distribution function of $\tilde{e}_1$ and $\tilde{e}_2$, respectively. Throughout the paper, random variables have a tilde (~) while their realizations do not.

Forward markets are incomplete in the sense that there are only forward contracts between the home and country 1's currencies. To hedge its exchange rate risk exposure, the firm sells (purchases if negative) h units of country 1's currency forward at a predetermined forward exchange rate. To focus on the hedging role, vis-à-vis the speculative role, of the currency forward contracts, we assume throughout the paper that the forward exchange rate is unbiased so that it is set equal to $\mathrm{E}(\tilde{e}_1)$.[6] The firm's random profit, denominated in the home currency, is given by

$$\tilde{\pi} = \tilde{e}_1 p_1 x_1 + \tilde{e}_2 p_2 x_2 - c(x_1 + x_2) + [\mathrm{E}(\tilde{e}_1) - \tilde{e}_1]h, \tag{1}$$

[3] It is noteworthy pointing out that the firm faces no exchange rate risk should p_1 and p_2 be denominated in the domestic currency. We assume local currency pricing because it is commonly observed in the real world. In the theoretical ground, Donnenfeld and Zilcha (1991) show that invoicing exports in the importers' currency entails a precommitment to prices so that quantities to be delivered are invariant to realized spot exchange rates. Furthermore, Friberg (1998) shows that setting prices in the importers' currency is optimal when the elasticity of exchange rate pass-through is less than unity, which is the dominant empirical finding in the literature (Menon, 1995).

[4] The assumption of imperfect arbitrage among national markets is supported by a number of empirical studies of the law of one price (see, e.g., Engel and Rogers, 1996, 2001; Parsley and Wei, 1996).

[5] An alternative way to model the exchange rate uncertainty is to apply the concept of information systems that are conditional probability density functions over a set of signals imperfectly correlated with $\tilde{e}$ (Drees and Eckwert, 2003; Eckwert and Zilcha, 2001, 2003). The advantage of this more general and realistic approach is that one can study the value of information by comparing the information content of different information systems. Since the focus of this paper is not on the value of information, we adopt a simpler structure to save notation.

[6] If the forward exchange rate is larger (smaller) than $\mathrm{E}(\tilde{e}_1)$, the positive (negative) risk premium induces the firm to speculate in the biased currency forward market by selling (purchasing) country 1's currency forward.

where $[\mathrm{E}(\tilde{e}_1) - \tilde{e}_1]h$ is the gain or loss due to the forward position, h.

The firm possesses a von Neumann-Morgenstern utility function, $u(\pi)$, defined over its home currency profit, π, with $u'(\pi) > 0$ and $u''(\pi) < 0$, indicating the presence of risk aversion.[7] The firm's ex-ante decision problem is to choose two levels of exports, x_1 and x_2, and a forward position, h, so as to maximize the expected utility of its home currency profit defined in equation (1):

$$\max_{x_1, x_2, h} \mathrm{E}[u(\tilde{\pi})], \tag{2}$$

subject to $x_1 \geq 0$ and $x_2 \geq 0$. The Kuhn-Tucker conditions for program (2) are given by

$$\mathrm{E}\{u'(\tilde{\pi}^*)[\tilde{e}_1 p_1 - c'(x_1^* + x_2^*)]\} \leq 0, \tag{3}$$

$$\mathrm{E}\{u'(\tilde{\pi}^*)[\tilde{e}_2 p_2 - c'(x_1^* + x_2^*)]\} \leq 0, \tag{4}$$

$$\mathrm{E}\{u'(\tilde{\pi}^*)[\mathrm{E}(\tilde{e}_1) - \tilde{e}_1]\} = 0, \tag{5}$$

where an asterisk (*) signifies an optimal level.[8] If $x_1^* > 0$, condition (3) holds with equality. Likewise, if $x_2^* > 0$, condition (4) holds with equality.

3. The benchmark case of complete forward markets

As a benchmark, we consider in this section the case that forward markets are complete, i.e., there are forward contracts between the home and country 1's currencies and between the home and country 2's currencies. Let h_i be the units of country i's currency sold

[7] For privately held, owner-managed firms, risk-averse behavior prevails. Even for publicly listed firms, managerial risk aversion (Stulz, 1984), corporate taxes (Smith and Stulz, 1985), costs of financial distress (Smith and Stulz, 1985), and capital market imperfections (Froot, Scharfstein, and Stein, 1993; Stulz, 1990) all imply a concave objective function for firms, thereby justifying the use of risk aversion as an approximation. See Tufano (1996) for evidence that managerial risk aversion is a rationale for corporate risk management in the gold mining industry.

[8] The second-order conditions for program (2) are satisfied given risk aversion and the strict convexity of $c(x)$.

(purchased if negative) forward by the firm at the unbiased forward exchange rate, $\mathrm{E}(\tilde{e}_i)$, where $i = 1$ and 2. The firm's home currency profit in this benchmark case is given by

$$\tilde{\pi}_b = \tilde{e}_1 p_1 x_1 + \tilde{e}_2 p_2 x_2 - c(x_1 + x_2) + [\mathrm{E}(\tilde{e}_1) - \tilde{e}_1]h_1 + [\mathrm{E}(\tilde{e}_2) - \tilde{e}_2]h_2, \tag{6}$$

where $[\mathrm{E}(\tilde{e}_1) - \tilde{e}_1]h_1 + [\mathrm{E}(\tilde{e}_2) - \tilde{e}_2]h_2$ is the net gain or loss due to the pair of forward positions, (h_1, h_2).

The firm's ex-ante decision problem is to choose two levels of exports, x_1 and x_2, and a pair of forward positions, (h_1, h_2), so as to maximize the expected utility of its home currency profit defined in equation (6):

$$\max_{x_1, x_2, h_1, h_2} \ \mathrm{E}[u(\tilde{\pi}_b)], \tag{7}$$

subject to $x_1 \geq 0$ and $x_2 \geq 0$. The Kuhn-Tucker conditions for program (7) are given by

$$\mathrm{E}\{u'(\tilde{\pi}_b^{**})[\tilde{e}_1 p_1 - c'(x_1^{**} + x_2^{**})]\} \leq 0, \tag{8}$$

$$\mathrm{E}\{u'(\tilde{\pi}_b^{**})[\tilde{e}_2 p_2 - c'(x_1^{**} + x_2^{**})]\} \leq 0, \tag{9}$$

$$\mathrm{E}\{u'(\tilde{\pi}_b^{**})[\mathrm{E}(\tilde{e}_1) - \tilde{e}_1]\} = 0, \tag{10}$$

$$\mathrm{E}\{u'(\tilde{\pi}_b^{**})[\mathrm{E}(\tilde{e}_2) - \tilde{e}_2]\} = 0, \tag{11}$$

where a double asterisk (**) signifies an optimal level. If $x_1^{**} > 0$, condition (8) holds with equality. Likewise, if $x_2^{**} > 0$, condition (9) holds with equality.

Proposition 1. *When forward markets are complete and unbiased, the competitive exporting firm's optimal levels of exports, x_1^{**} and x_2^{**}, are independent of its risk attitude and of the underlying exchange rate uncertainty, and its optimal pair of forward positions, (h_1^{**}, h_2^{**}), satisfies that $h_1^{**} = p_1 x_1^{**}$ and $h_2^{**} = p_2 x_2^{**}$.*

(i) *If* $\mathrm{E}(\tilde{e}_1)p_1 > \mathrm{E}(\tilde{e}_2)p_2$, *the firm exports exclusively to country 1 with x_1^{**} that solves $c'(x_1^{**}) = \mathrm{E}(\tilde{e}_1)p_1$ and $x_2^{**} = 0$.*

(ii) *If* $\mathrm{E}(\tilde{e}_2)p_2 > \mathrm{E}(\tilde{e}_1)p_1$, *the firm exports exclusively to country 2 with* x_2^{**} *that solves* $c'(x_2^{**}) = \mathrm{E}(\tilde{e}_2)p_2$ *and* $x_1^{**} = 0$.

(iii) *If* $\mathrm{E}(\tilde{e}_1)p_1 = \mathrm{E}(\tilde{e}_2)p_2$, *the firm is indifferent between exporting to country 1 and exporting to country 2 with* $x_1^{**} + x_2^{**}$ *that solves* $c'(x_1^{**} + x_2^{**}) = \mathrm{E}(\tilde{e}_1)p_1$.

To see the intuition of Proposition 1, we recast equation (6) as

$$\tilde{\pi}_b = \mathrm{E}(\tilde{e}_1)p_1x_1 + \mathrm{E}(\tilde{e}_2)p_2x_2 - c(x_1 + x_2)$$

$$+[\mathrm{E}(\tilde{e}_1) - \tilde{e}_1](h_1 - p_1x_1) + [\mathrm{E}(\tilde{e}_2) - \tilde{e}_2](h_2 - p_2x_2), \tag{12}$$

Inspection of equation (12) reveals that the firm could have completely eliminated its exchange rate risk exposure had it chosen $h_1 = p_1x_1$ and $h_2 = p_2x_2$ within its own discretion. Alternatively put, the degree of exchange rate risk exposure to be assumed by the firm should be totally unrelated to its production and export decisions. The firm as such behaves as if it is risk neutral so that its optimal levels of exports, x_1^{**} and x_2^{**}, maximize $\mathrm{E}(\tilde{e}_1)p_1x_1 + \mathrm{E}(\tilde{e}_2)p_2x_2 - c(x_1 + x_2)$. When $\mathrm{E}(\tilde{e}_1)p_1 > (<)\ \mathrm{E}(\tilde{e}_2)p_2$, the "risk-neutral" firm finds it optimal to export exclusively to country 1 (2). The firm is indifferent between exporting to country 1 and country 2 only when $\mathrm{E}(\tilde{e}_1)p_1 = \mathrm{E}(\tilde{e}_2)p_2$. Since the unbiased currency forward markets offer actuarially fair "insurance" to the firm, the risk-averse firm optimally opts for full insurance by choosing $h_1^{**} = p_1x_1^{**}$ and $h_2^{**} = p_2x_2^{**}$, which completely eliminates its exchange rate risk exposure. The results of Proposition 1 are simply the celebrated separation and full-hedging theorems emanated from the literature on the competitive exporting firm under exchange rate uncertainty.

4. Optimal export and hedging decisions

In this section, we resume the firm's original decision problem when forward markers are incomplete. To facilitate the exposition, we consider first the hypothetical case that the firm does not export to country 1, i.e., $x_1 \equiv 0$. In this case, the first-order conditions for program (2) become

$$\mathrm{E}\{u'(\tilde{\pi}^0)[\tilde{e}_2 p_2 - c'(x_2^0)]\} = 0, \tag{13}$$

$$\mathrm{E}\{u'(\tilde{\pi}^0)[\mathrm{E}(\tilde{e}_1) - \tilde{e}_1]\} = 0, \tag{14}$$

where $\tilde{\pi}^0 = \tilde{e}_2 p_2 x_2^0 - c(x_2^0) + [\mathrm{E}(\tilde{e}_1) - \tilde{e}_1]h^0$, and a nought (^{0}) indicates an optimal level.

Using the covariance operator, we can write equation (13) as[9]

$$\mathrm{E}(\tilde{e}_2)p_2 - c'(x_2^0) = -\frac{\mathrm{Cov}[u'(\tilde{\pi}^0), \tilde{e}_2]p_2}{\mathrm{E}[u'(\tilde{\pi}^0)]}. \tag{15}$$

Since equation (14) implies that $\mathrm{Cov}[u'(\tilde{\pi}^0), \tilde{e}_1] = 0$, we have

$$\mathrm{Cov}[u'(\tilde{\pi}^0), \tilde{\pi}^0] = \mathrm{Cov}[u'(\tilde{\pi}^0), \tilde{e}_2]p_2 x_2^0 - \mathrm{Cov}[u'(\tilde{\pi}^0), \tilde{e}_1]h^0 = \mathrm{Cov}[u'(\tilde{\pi}^0), \tilde{e}_2]p_2 x_2^0. \tag{16}$$

Risk aversion implies that $\mathrm{Cov}[u'(\tilde{\pi}^0), \tilde{\pi}^0] < 0$. It then follows from equations (15) and (16) that $c'(x_2^0) < \mathrm{E}(\tilde{e}_2)p_2$.

Proposition 2. *The competitive exporting firm has access to the unbiased forward market between the home and country 1's currencies.*

(i) *If* $\mathrm{E}(\tilde{e}_1)p_1 \geq \mathrm{E}(\tilde{e}_2)p_2$, *the firm exports exclusively to country 1 with* x_1^* *that solves* $c'(x_1^*) = \mathrm{E}(\tilde{e}_1)p_1$ *and* $x_2^* = 0$, *and optimally opts for the forward position,* $h^* = p_1 x_1^*$.

(ii) *If* $\mathrm{E}(\tilde{e}_2)p_2 > c'(x_2^0) \geq \mathrm{E}(\tilde{e}_1)p_1$, *the firm exports exclusively to country 2 with* $x_2^* = x_2^0$ *and* $x_1^* = 0$, *and optimally opts for the forward position,* $h^* = h^0$, *where* x_2^0 *and* h^0 *solve equations (13) and (14) simultaneously.*

(iii) *If* $\mathrm{E}(\tilde{e}_2)p_2 > \mathrm{E}(\tilde{e}_1)p_1 > c'(x_2^0)$, *the firm optimally produces the output level,* $x_1^* + x_2^*$, *that solves* $c'(x_1^* + x_2^*) = \mathrm{E}(\tilde{e}_1)p_1$, *and exports to both countries, i.e.,* $x_1^* > 0$ *and* $x_2^* > 0$. *The optimal levels of exports,* x_1^* *and* x_2^*, *and the optimal forward position,* h^*, *are uniquely determined by solving conditions (3)–(5) with equality simultaneously.*

[9]For any two random variables, $\tilde{x}$ and $\tilde{y}$, we have $\mathrm{Cov}(\tilde{x}, \tilde{y}) = \mathrm{E}(\tilde{x}\tilde{y}) - \mathrm{E}(\tilde{x})\mathrm{E}(\tilde{y})$.

To see the intuition of Proposition 1, we recast equation (1) as

$$\tilde{\pi} = \mathrm{E}(\tilde{e}_1)p_1x_1 - c(x_1 + x_2) + \tilde{e}_2p_2x_2 + [\mathrm{E}(\tilde{e}_1) - \tilde{e}_1](h - p_1x_1). \tag{17}$$

Given the forward hedge via the contracts between the home and country 1's currencies, it is evident from equation (17) that the marginal revenue from exports to country 1 is locked in at the deterministic level, $\mathrm{E}(\tilde{e}_1)p_1$. Since the marginal revenue from exports to country 2 is $\tilde{e}_2p_2$ that is stochastic, the risk-averse firm sells exclusively in country 1 if the expected marginal revenue from exports to country 2 does not exceed the deterministic marginal revenue from exports to country 1, i.e., $\mathrm{E}(\tilde{e}_2)p_2 \leq \mathrm{E}(\tilde{e}_1)p_1$. In this case, the separation and full-hedging theorems apply in that x_1^* solves $c'(x_1^*) = \mathrm{E}(\tilde{e}_1)p_1$ and $h^* = p_1x_1^*$. In contrast to part (iii) of Proposition 1, the firm strictly prefers to export to country 1 when $\mathrm{E}(\tilde{e}_1)p_1 = \mathrm{E}(\tilde{e}_2)p_2$ given that the forward contracts between the home and country 2's currencies are missing.

If $\mathrm{E}(\tilde{e}_2)p_2 > c'(x_2^0) \geq \mathrm{E}(\tilde{e}_1)p_1$, the deterministic marginal revenue from exports to country 1 is not enough to cover the marginal cost of production at $x = x_2^0$, the optimal output level should the firm sell exclusively in country 2. In this case, it does not pay for the firm to sell in country 1 so that $x_1^* = 0$ and $x_2^* = x_2^0$. The optimal level of exports to country 2, x_2^0, and the optimal forward position, h_0, are uniquely determined by solving equations (13) and (14) simultaneously, from which we can see that neither the separation theorem nor the full-hedging theorem holds. Since $c'(x_2^0) < \mathrm{E}(\tilde{e}_2)p_2 = c'(x_2^{**})$, it follows from the strict convexity of $c(x)$ that $x_2^0 < x_2^{**}$. In response to forward market incompleteness, the firm optimally cuts down its production to limit its exposure to the exchange rate risk.

If $\mathrm{E}(\tilde{e}_2)p_2 > \mathrm{E}(\tilde{e}_1)p_1 > c'(x_2^0)$, selling in country 1 is optimal and the firm equates the marginal cost of production to the deterministic marginal revenue from exports to country 1. The optimal levels of exports, x_1^* and x_2^*, and the optimal forward position, h^*, are uniquely determined by solving conditions (3)–(5) with equality simultaneously. Since $c'(x_1^* + x_2^*) = \mathrm{E}(\tilde{e}_1)p_1 < \mathrm{E}(\tilde{e}_2)p_2 = c'(x_2^{**})$, it follows from the strict convexity of $c(x)$ that $x_1^* + x_2^* < x_2^{**}$. Thus, the firm produces less than the optimal level under compete forward markets. While the firm's optimal output level, $x_1^* + x_2^*$, is independent of its risk attitude and of the underlying exchange rate uncertainty, the optimal levels of exports, x_1^* and x_2^*, are not, rendering the partial collapse of the separation theorem. Furthermore, the firm may or may not opt for a full-hedge, i.e., h^* may or may not be equal to $p_1 x_1^*$, without knowing the specific joint probability distribution function of $\tilde{e}_1$ and $\tilde{e}_2$. Thus, the full-hedging theorem fails to hold.

As shown in Proposition 2, we can explicitly characterize the optimal forward position, h^*, only in the case that $\mathrm{E}(\tilde{e}_1)p_1 \geq \mathrm{E}(\tilde{e}_2)p_2$, under which the full-hedging theorem holds. In the case that $\mathrm{E}(\tilde{e}_1)p_1 < \mathrm{E}(\tilde{e}_2)p_2$, we need to impose some tractable dependence structure on $\tilde{e}_1$ and $\tilde{e}_2$ so as to derive some concrete results of h^*. To this end, we adopt the following concept of bivariate dependence proposed by Lehmann (1966).

Definition 1. *Let $G(e_1)$ be the cumulative distribution function (CDF) of $\tilde{e}_1$ over support $[\underline{e}_1, \overline{e}_1]$, where $0 \leq \underline{e}_1 < \overline{e}_1 \leq \infty$. Given a realized value of $\tilde{e}_1$, let $F(e_2|e_1)$ be the conditional CDF of $\tilde{e}_2$ over support $[\underline{e}_2, \overline{e}_2]$, where $0 \leq \underline{e}_2 < \overline{e}_2 \leq \infty$. We say that $\tilde{e}_2$ is positively (negatively) regression dependent on $\tilde{e}_1$ if, and only if, the conditional CDF of $\tilde{e}_2$ improves (deteriorates) in the sense of first-degree stochastic dominance as the realized value of $\tilde{e}_1$ increases; i.e., $F_{e_1}(e_2|e_1) \leq (\geq)\ 0$ for all $(e_1, e_2) \in [\underline{e}_1, \overline{e}_1] \times [\underline{e}_2, \overline{e}_2]$, where the subscript, e_1, denotes a partial derivative.*

The positive (negative) regression dependence has the intuition that $\tilde{e}_1$ and $\tilde{e}_2$ tend to move in the same direction (opposite directions). Using Chebyshev inequality, Shea (1979) shows that Definition 1 implies that $\tilde{e}_1$ and $\tilde{e}_2$ are positively (negatively) correlated (the converse is not true though). For example, if $\tilde{e}_1$ and $\tilde{e}_2$ are normally distributed, they will be positively or negatively regression dependent according to whether the correlation coefficient is positive or negative, respectively.[10] If $\tilde{e}_1$ and $\tilde{e}_2$ are independent, we have $F(e_2|e_1)$ to be invariant to all $e_1 \in [\underline{e}_1, \overline{e}_1]$.

[10] Aboudi and Thon (1995) argue that the concept of regression dependence is the most suitable concept of bivariate dependence for applications to the theory of choice under multiple sources of uncertainty. See also Wong (1996).

Proposition 3. *Suppose that the competitive exporting firm has access to the unbiased forward market between the home and country 1's currencies and that* $\mathrm{E}(\tilde{e}_1)p_1 < \mathrm{E}(\tilde{e}_2)p_2$*. The firm's optimal forward position,* h^**, satisfies that* $h^* > (<)\ p_1x_1^*$ *if* $\tilde{e}_2$ *is positively (negatively) regression dependent on* $\tilde{e}_1$*, and that* $h^* = p_1x_1^*$ *if* $\tilde{e}_1$ *and* $\tilde{e}_2$ *are independent.*

To see the intuition of Proposition 3, we use the covariance operator to recast equation (5) as

$$\mathrm{Cov}[u'(\tilde{\pi}^*), \tilde{e}_1] = 0. \tag{18}$$

Since covariances can be interpreted as marginal variances, inspection of equation (18) reveals that the optimal forward position, h^*, minimizes the variance of the marginal utility across different realizations of $\tilde{e}_1$. If $\tilde{e}_1$ and $\tilde{e}_2$ are positively (negatively) regression dependent, we have

$$\mathrm{Cov}\{u'[\mathrm{E}(\tilde{e}_1)p_1x_1^* + \tilde{e}_2p_2x_2^* - c(x_1^* + x_2^*)], \tilde{e}_1\} < (>)\ 0, \tag{19}$$

at $h^* = p_1x_1^*$ since $u''(\pi) < 0$. To further reduce the variability of its marginal utility, it follows from inequality (19) that the firm opts for $h^* > (<)\ p_1x_1^*$. If $\tilde{e}_1$ and $\tilde{e}_2$ are independent, the left-hand side of inequality (19) vanishes and thus $h^* = p_1x_1^*$.

5. Conclusions

In this paper, we have examined the behavior of the competitive firm that exports to two foreign countries under multiple sources of exchange rate uncertainty. Forward market incompleteness is introduced by assuming that one foreign country's currency has no derivatives contracts written on it, while the other foreign country's currency has a forward market to which the firm has access.

Within this framework, we have shown that the separation theorem holds when the firm optimally exports to the foreign country with the currency forward market. The full-hedging theorem holds when the firm either exports exclusively to the foreign country with the currency forward market or when the relevant spot exchange rates are independent. In the case that the relevant spot exchange rates are positively (negatively) correlated in the sense of regression dependence, we have shown that the firm optimally opts for a short (long) forward position for cross-hedging purposes.

Cross-hedging is important because it expands the opportunity set of hedging alternatives. Given the fact that currency forward markets are not readily available in many less developed countries and in many of the newly industrializing countries of Latin America and Asia Pacific, exporting firms that expose themselves to currencies of these countries should continue to regard cross-hedging as a major risk management technique for the reduction of their exchange rate risk exposure.

Appendix

A. Proof of Proposition 1

Multiplying p_1 to equation (10) and substituting the resulting equation into inequality (8) yields

$$\mathrm{E}(\tilde{e}_1)p_1 - c'(x_1^{**} + x_2^{**}) \leq 0, \tag{A.1}$$

since $u'(\pi) > 0$. Likewise, multiplying p_2 to equation (11) and substituting the resulting equation into inequality (9) yields

$$\mathrm{E}(\tilde{e}_2)p_2 - c'(x_1^{**} + x_2^{**}) \leq 0. \tag{A.2}$$

If $\mathrm{E}(\tilde{e}_1)p_1 > \mathrm{E}(\tilde{e}_2)p_2$, inequality (A.1) holds with equality and inequality (A.2) holds as a strict inequality, implying that x_1^{**} solves $c'(x_1^{**}) = \mathrm{E}(\tilde{e}_1)p_1$ and $x_2^{**} = 0$. Likewise, if $\mathrm{E}(\tilde{e}_1)p_1 < \mathrm{E}(\tilde{e}_2)p_2$, we have x_2^{**} that solves $c'(x_2^{**}) = \mathrm{E}(\tilde{e}_2)p_2$ and $x_1^{**} = 0$. The firm is indifferent between exporting to country 1 and country 2 only when $\mathrm{E}(\tilde{e}_1)p_1 = \mathrm{E}(\tilde{e}_2)p_2$. In this case, $x_1^{**} + x_2^{**}$ solves $c'(x_1^{**} + x_2^{**}) = \mathrm{E}(\tilde{e}_1)p_1$.

Using the covariance operator, we can write equations (10) and (11) as

$$\mathrm{Cov}[u'(\tilde{\pi}_b^{**}), \tilde{e}_1] = 0, \tag{A.3}$$

$$\mathrm{Cov}[u'(\tilde{\pi}_b^{**}), \tilde{e}_2] = 0. \tag{A.4}$$

If $h_1^{**} = p_1 x_1^{**}$ and $h_2^{**} = p_2 x_2^{**}$, it follows from equation (6) that $\tilde{\pi}_b^{**} = \mathrm{E}(\tilde{e}_1)p_1 x_1^{**} + \mathrm{E}(\tilde{e}_2)p_2 x_2^{**} - c(x_1^{**} + x_2^{**})$, which is invariant to different realizations of $\tilde{e}_1$ and $\tilde{e}_2$. Hence, $h_1^{**} = p_1 x_1^{**}$ and $h_2^{**} = p_2 x_2^{**}$ indeed solve equations (A.3) and (A.4) simultaneously. □

B. Proof of Proposition 2

We formulate program (2) as a two-stage optimization problem. In the first stage, the firm chooses the optimal level of exports to country 1, $x_1(x_2)$, and the optimal forward position, $h(x_2)$, for a given level of exports to country 2, x_2. In the second stage, the firm chooses the optimal level of exports to country 2, x_2^*, taking $x_1(x_2)$ and $h(x_2)$ as given. The complete solution to program (2) is thus x_2^*, $x_1^* = x_1(x_2^*)$, and $h^* = h(x_2^*)$. The solution to the first-stage optimization problem, $\{x_1(x_2), h(x_2)\}$, satisfies the following Kuhn-Tucker conditions:

$$\mathrm{E}\Big\{u'[\tilde{\pi}(x_2)]\{\tilde{e}_1 p_1 - c'[x_1(x_2) + x_2]\}\Big\} \leq 0, \tag{A.5}$$

$$\mathrm{E}\{u'[\tilde{\pi}(x_2)][\mathrm{E}(\tilde{e}_1) - \tilde{e}_1]\} = 0, \tag{A.6}$$

where $\tilde{\pi}(x_2) = \tilde{e}_1 p_1 x_1(x_2) + \tilde{e}_2 p_2 x_2 - c[x_1(x_2) + x_2] + [\mathrm{E}(\tilde{e}_1) - \tilde{e}_1]h(x_2)$. If $x_1(x_2) > 0$, condition (A.5) holds with equality. Multiplying p_1 to equation (A.6) and adding the resulting equation to condition (A.5) yields

$$\mathrm{E}(\tilde{e}_1)p_1 - c'[x_1(x_2) + x_2] \leq 0, \tag{A.7}$$

since $u'(\pi) > 0$. For x_2 sufficiently small in that $c'(x_2) < \mathrm{E}(\tilde{e}_1)p_1$, it follows that $x_1(x_2) > 0$ and inequality (A.7) holds with equality. Thus, when $x_2 = 0$, we have

$$\mathrm{E}(\tilde{e}_1)p_1 - c'[x_1(0)] = 0. \tag{A.8}$$

In this case, $h(0) = p_1x_1(0)$ solves equation (A.6) since $\pi(0) = \mathrm{E}(\tilde{e}_1)p_1x_1(0) - c[x_1(0)]$ is non-stochastic. Let EU be the objective function of program (2) with $x_1 = x_1(x_2)$ and $h = h(x_2)$. Totally differentiating EU with respect to x_2, using the envelope theorem, and evaluating the resulting derivative at $x_2 = 0$ yields

$$\left.\frac{\mathrm{d}EU}{\mathrm{d}x_2}\right|_{x_2=0} = u'[\pi(0)]\{\mathrm{E}(e_2)p_2 - c'[x_1(0)]\}. \tag{A.9}$$

Substituting equation (A.8) into the right-hand side of equation (A.9) yields

$$\left.\frac{\mathrm{d}EU}{\mathrm{d}x_2}\right|_{x_2=0} = u'[\pi(0)][\mathrm{E}(\tilde{e}_2)p_2 - \mathrm{E}(\tilde{e}_1)p_1]. \tag{A.10}$$

If $\mathrm{E}(\tilde{e}_1)p_1 \geq \mathrm{E}(\tilde{e}_2)p_2$, equation (A.10) implies that $x_2^* = 0$. We then know from equation (A.8) that x_1^* solves $c'(x_1^*) = \mathrm{E}(\tilde{e}_1)p_1$ and $h^* = p_1x_1^*$. This proves part (i) of Proposition 2.

If $\mathrm{E}(\tilde{e}_2)p_2 > \mathrm{E}(\tilde{e}_1)p_1$, equation (A.10) implies that $x_2^* > 0$. In this case, inequality (4) holds with equality. Let us reformulate program (2) as a two-stage optimization problem. In the first stage, the firm chooses the optimal amount of exports to country 2, $x_2(x_1)$, and the optimal forward position, $h(x_1)$, for a given amount of exports to country 1, x_1. In the second stage, the firm chooses the optimal amount of exports to country 1, x_1^*, taking $x_2(x_1)$ and $h(x_1)$ as given. The complete solution to program (2) is thus x_1^*, $x_2^* = x_2(x_1^*)$, and $h^* = h(x_1^*)$. The solution to the first-stage optimization problem, $\{x_2(x_1), h(x_1)\}$, satisfies the following first-order conditions:

$$\mathrm{E}\Big\{u'[\tilde{\pi}(x_1)]\{\tilde{e}_2p_2 - c'[x_1 + x_2(x_1)]\}\Big\} = 0, \tag{A.11}$$

$$\mathrm{E}\{u'[\tilde{\pi}(x_1)][\mathrm{E}(\tilde{e}_1) - \tilde{e}_1]\} = 0, \tag{A.12}$$

where $\tilde{\pi}(x_1) = \tilde{e}_1p_1x_1 + \tilde{e}_2p_2x_2(x_1) - c[x_1 + x_2(x_1)] + [\mathrm{E}(\tilde{e}_1) - \tilde{e}_1]h(x_1)$. Let EU be the objective function of program (2) with $x_2 = x_2(x_1)$ and $h = h(x_1)$. Totally differentiating

EU with respect to x_1, using the envelope theorem, and evaluating the resulting derivative at $x_1 = 0$ yields

$$\left.\frac{\mathrm{d}EU}{\mathrm{d}x_1}\right|_{x_1=0} = \mathrm{E}\{u'(\tilde{\pi}^0)[\tilde{e}_1 p_1 - c'(x_2^0)]\}, \tag{A.13}$$

where x_2^0 and h^0 are defined in equations (13) and (14). Substituting equation (14) into the right-hand side of equation (A.13) yields

$$\left.\frac{\mathrm{d}EU}{\mathrm{d}x_1}\right|_{x_1=0} = \mathrm{E}[u'(\tilde{\pi}^0)][\mathrm{E}(\tilde{e}_1)p_1 - c'(x_2^0)]. \tag{A.14}$$

If $c'(x_2^0) \geq \mathrm{E}(\tilde{e}_1)p_1$, equation (A.14) implies that $x_1^* = 0$. Thus, in this case we have $x_2^* = x_2^0$ and $h^* = h^0$. This proves part (ii) of Proposition 1.

Finally, if $c'(x_2^0) < \mathrm{E}(\tilde{e}_1)p_1$, equation (A.14) implies that $x_1^* > 0$. In this case, inequality (3) holds with equality:

$$\mathrm{E}\{u'(\tilde{\pi}^*)[\tilde{e}_1 p_1 - c'(x_1^* + x_2^*)]\} = 0. \tag{A.15}$$

Multiplying p_1 to equation (5) and adding the resulting equation to equation (A.15) yields

$$c'(x_1^* + x_2^*) = \mathrm{E}(\tilde{e}_1)p_1, \tag{A.16}$$

since $u'(\pi) > 0$. Hence, the optimal output level, $x_1^* + x_2^*$, solves equation (A.16). This proves part (iii) of Proposition 2. $\square$

C. Proof of Proposition 3

Rewrite equation (5) as

$$\int_{\underline{e}_1}^{\overline{e}_1} \{M(e_1) - M[\mathrm{E}(\tilde{e}_1)]\}[\mathrm{E}(\tilde{e}_1) - e_1]\ \mathrm{d}G(e_1) = 0, \tag{A.17}$$

where $M(e_1)$ is the conditional expectation of $u'(\pi^*)$ with respect to $F(e_2|e_1)$:

$$M(e_1) = \int_{\underline{e}_2}^{\overline{e}_2} u'\{e_1p_1x_1^* + e_2p_2x_2^* - c(x_1^* + x_2^*) + [\mathrm{E}(\tilde{e}_1) - e_1]h^*\}\ \mathrm{d}F(e_2|e_1). \tag{A.18}$$

Using integration by parts, we can write equation (A.18) as

$$M(e_1) = u'\{e_1p_1x_1^* + \overline{e}_2p_2x_2^* - c(x_1^* + x_2^*) + [\mathrm{E}(\tilde{e}_1) - e_1]h^*\}$$

$$- \int_{\underline{e}_2}^{\overline{e}_2} u''\{e_1p_1x_1^* + e_2p_2x_2^* - c(x_1^* + x_2^*) + [\mathrm{E}(\tilde{e}_1) - e_1]h^*\}p_2x_2^*F(e_2|e_1)\ \mathrm{d}e_2. \tag{A.19}$$

Partially differentiating equation (A.19) with respect to e_1 yields

$$M'(e_1) = \int_{\underline{e}_2}^{\overline{e}_2} u''\{e_1p_1x_1^* + e_2p_2x_2^* - c(x_1^* + x_2^*) + [\mathrm{E}(\tilde{e}_1) - e_1]h^*\}(p_1x_1^* - h^*)\ \mathrm{d}F(e_2|e_1)$$

$$- \int_{\underline{e}_2}^{\overline{e}_2} u''\{e_1p_1x_1^* + e_2p_2x_2^* - c(x_1^* + x_2^*) + [\mathrm{E}(\tilde{e}_1) - e_1]h^*\}p_2x_2^*F_{e_1}(e_2|e_1)\ \mathrm{d}e_2. \tag{A.20}$$

Consider the case that $\tilde{e}_2$ is positively (negatively) regression dependent on $\tilde{e}_1$. Suppose that $h^* \leq (\geq)\ p_1x_1^*$. Equation (A.20) implies that $M'(e_1) \leq (\geq)\ 0$. The left-hand side of equation (A.17) becomes positive (negative), a contradiction. Hence, we have $h^* > (<)\ p_1x_1^*$ if $\tilde{e}_2$ is positively (negatively) regression dependent on $\tilde{e}_1$.

Now, consider the case that $\tilde{e}_1$ and $\tilde{e}_2$ are independent. From equation (A.20), $M'(e_1) = 0$ at $h^* = p_1x_1^*$ so that equation (A.17) is satisfied. Hence, we have $h^* = p_1x_1^*$ if $\tilde{e}_1$ and $\tilde{e}_2$ are Independent. $\square$

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