Foreign Direct Investment and Forward Hedging [*]

Kit Pong WONG [†]

University of Hong Kong

December 2005

This paper examines the behavior of a risk-averse multinational firm (MNF) making investment in a foreign country under exchange rate uncertainty. To hedge the exchange rate risk, the MNF has access to an unbiased currency forward market. Foreign direct investment (FDI) is irreversible and sequential in that the MNF can acquire additional capital after the exchange rate uncertainty is completely resolved. The MNF as such possesses a real (call) option that is rationally exercised whenever the foreign currency has been substantially appreciated relative to the domestic currency. We show that the MNF's optimal initial level of sequential FDI is always lower than that of lumpy FDI, while the expected optimal aggregate level of sequential FDI can be higher or lower than that of lumpy FDI. We further show that the presence of the currency forward market improves the MNF's incentives to make FDI, both ex ante and ex post.

JEL classification: D81; F23; F31

Keywords: Foreign direct investment; Real options; Currency forwards

[*]I gratefully acknowledge financial support from a grant provided by the University Grants Committee of the Hong Kong Special Administrative Region, China (Project No. AoE/H-05/99). I would like to thank Ike Mathur (the editor) and an anonymous referee for their helpful comments and suggestions. The usual disclaimer applies.

[†]*Correspondence to:* Kit Pong Wong, School of Economics and Finance, University of Hong Kong, Pokfulam Road, Hong Kong. Tel.: (852) 2859-1044, fax: (852) 2548-1152, e-mail: kpwong@econ.hku.hk (K.P. Wong).

Foreign direct investment and forward hedging

Abstract

This paper examines the behavior of a risk-averse multinational firm (MNF) making investment in a foreign country under exchange rate uncertainty. To hedge the exchange rate risk, the MNF has access to an unbiased currency forward market. Foreign direct investment (FDI) is irreversible and sequential in that the MNF can acquire additional capital after the exchange rate uncertainty is completely resolved. The MNF as such possesses a real (call) option that is rationally exercised whenever the foreign currency has been substantially appreciated relative to the domestic currency. We show that the MNF's optimal initial level of sequential FDI is always lower than that of lumpy FDI, while the expected optimal aggregate level of sequential FDI can be higher or lower than that of lumpy FDI. We further show that the presence of the currency forward market improves the MNF's incentives to make FDI, both ex ante and ex post.

JEL classification: D81; F23; F31

Keywords: Foreign direct investment; Real options; Currency forwards

1. Introduction

The effect of a currency forward market on the behavior of a risk-averse multinational firm (MNF) under exchange rate uncertainty has been extensively studied in the literature (see, e.g., Adam-Müller, 1997; Broll, 1992; Broll and Zilcha, 1992; Chang and Wong, 2003; Lien and Wang, 2006; Wong, 2003a, 2003b). The typical scenario is that the MNF makes its investment and hedging decisions simultaneously prior to the resolution of the exchange rate uncertainty. Two notable results emanate. First, the separation theorem states that the MNF's optimal investment decision is affected neither by its risk attitude nor by the underlying exchange rate uncertainty in the presence of the currency forward market. Second, the full-hedging theorem states that the MNF optimally opts for a full-hedge to completely eliminate its exchange rate risk exposure should the currency forward market be unbiased.

As argued by Kogut (1983), foreign direct investment (FDI) is a sequential process that determines the volume and direction of resources transferred across borders. The ability of MNFs to arbitrage institutional restrictions (e.g., tax codes, relative exchange rates, and remittance forms) creates a string of options written on contingent outcomes. MNFs as such are best viewed as a collection of valuable options that permit discretionary choices among alternative real economic activities and financial flows from one country to the other.

Regarding FDI as a sequential process, we depart from the extant literature by allowing the risk-averse MNF to make sequential, instead of lumpy, investment in a foreign country. After the exchange rate uncertainty has been completely resolved, the MNF has the right, but not the obligation, to alter its level of FDI. Specifically, the MNF is eligible for purchasing additional capital in the domestic country at a higher per-unit price of capital. This reflects the fact that FDI is by and large irreversible and costly expandable.[1]

Within this framework, we show that neither the separation theorem nor the full-hedging theorem holds. The flexibility of making sequential FDI, vis-à-vis lumpy FDI, is shown to offer the MNF a real (call) option that is rationally exercised whenever the foreign currency has been substantially appreciated relative to the domestic currency. Since the value of a call option increases with a decrease in its exercise price (Merton, 1973), the MNF has incentives to cut down its initial level of FDI so as to make the ex-post exercise of the real option more likely. The optimal initial level of sequential FDI as such is unambiguously lower than that of lumpy FDI. The expected optimal aggregate level of sequential FDI, however, can be higher or lower than that of lumpy FDI. Anticipating that its ultimate level of FDI is likely to be adjusted upward should the realized spot exchange rate be favorable, the MNF optimally opts for an over-hedge in the unbiased currency forward market.

Using samples of U.S. multinational corporations, Allayannis, Ihrig, and Weston (2001), Kim, Mathur, and Nam (2006), and Pantzalis, Simkins, and Laux (2001) find that operational hedges serve as real options and are effective when used in combination with financial hedges in reducing exchange rate risk exposure and enhancing firm value. Furthermore, Al-

[1]Asset specificity, information asymmetry, and government regulations are plausible reasons why FDI is irreversible (Dixit and Pindyck, 1994).

layannis, Ihrig, and Weston (2001) and Kim, Mathur, and Nam (2006) document that operational hedges and financial hedges are complements rather than substitutes. Since the full-hedging theorem applies when the MNF is restricted to make lumpy FDI, this complementary nature of operational and financial hedging strategies is consistent with our over-hedging result in the case of sequential FDI.

If the risk-averse MNF is banned from trading in the unbiased currency forward market, we show that the MNF's ex-ante and ex-post incentives to make FDI are reduced with either a global or a marginal increase in risk aversion. Since a financial hedge is absent, the MNF has to rely on an operational hedge via lowing its FDI. We further show that an increase in the fixed or setup cost incurred by the MNF gives rise to the same perverse effect on FDI should the MNF's risk preferences exhibit the reasonable property of decreasing absolute risk aversion. Given that the change in the fixed or setup cost may be due to a change in the investment tax credits offered by the host government, or due to a change in the severity of entry barriers in the host country, FDI flows are expected to react in a predictable manner when these government policies and market conditions shift over time. These implications are largely consistent with the empirical findings of Hines (2001) and Anand and Kogut (1997).

If the risk-averse MNF has access to the unbiased currency forward market, we show that risk aversion has no effect on the expected marginal return to the initial level of FDI, but has a negative effect on the option value of waiting to make subsequent FDI. The former is due to the spanning property that arises from the tradability of the random spot exchange rate in the unbiased currency forward market. The latter is due to the non-tradability of the real option embedded in sequential FDI so that spanning is not possible, making the MNF's risk preferences impact negatively on the pricing of the option in this incomplete market context. The MNF's ex-ante and ex-post incentives to make FDI are therefore improved as compared to those under risk neutrality. This implies immediately that forward hedging promotes FDI, a result consistent with the complementary nature of operational and financial hedging strategies as empirically documented by Allayannis, Ihrig, and Weston (2001) and Kim, Mathur, and Nam (2006).

The rest of this paper is organized as follows. Section 2 delineates our model of the risk-averse MNF that makes sequential FDI decisions in response to the intertemporal resolution of the exchange rate uncertainty. Section 3 compares the MNF's optimal level of sequential FDI with that of lumpy FDI. Section 4 examines the MNF's optimal sequential FDI decisions when the unbiased currency forward market is not available to the MNF for hedging purposes. Section 5 resumes the MNF's access to the unbiased currency forward market and shows that forward hedging improves the MNF's ex-ante and ex-post incentives to make FDI. The final section concludes.

2. The model

Consider a multinational firm (MNF) that invests in a foreign country under exchange rate uncertainty. There is one period with three dates, indexed by $t = 0$, 1, and 2. The prevailing spot exchange rate at $t = 2$, which is denoted by $\tilde{e}$ and is expressed in units of the domestic currency per unit of the foreign currency, is uncertain at $t = 0$.[2] The MNF regards $\tilde{e}$ as a positive random variable distributed according to a known cumulative distribution function, $F(e)$, over support $[\underline{e}, \overline{e}]$, where $0 \leq \underline{e} < \overline{e} \leq \infty$.[3] The exchange rate uncertainty, however, is completely resolved at $t = 1$, at which time the true realization of $\tilde{e}$ is publicly observed. The riskless rate of interest is known and constant for the period. To simplify notation, we henceforth suppress the interest factors by compounding all cash flows to their future values at $t = 2$.

To begin, the MNF incurs a fixed cost, c, for the access to a project in the foreign country. If the MNF makes foreign direct investment (FDI) of k units of capital that are acquired in the home country, the project yields a deterministic cash flow of $f(k)$ at $t = 2$,

[2] Throughout the paper, random variables have a tilde ($\tilde{}$) while their realizations do not.

[3] An alternative way to model the exchange rate uncertainty is to apply the concept of information systems that are conditional cumulative distribution functions over a set of signals imperfectly correlated with $\tilde{e}$ (see Broll and Eckwert, 2006; Drees and Eckwert, 2003; Eckwert and Zilcha, 2001, 2003). The advantage of this more general and realistic approach is that one can study the value of information by comparing the information content of different information systems. Since the focus of this paper is not on the value of information, we adopt a simpler structure to save notation.

where $f(k)$ is denominated in the foreign currency with $f(0) = 0$, $f'(k) > 0$, $f''(k) < 0$, $\lim_{k\to 0} f'(k) = \infty$, and $\lim_{k\to\infty} f'(k) = 0$. FDI is irreversible and sequential. Succinctly, at $t = 0$, the MNF acquires k_0 units of capital at a known per-unit price, p_0, in the home country, where p_0 is denominated in the domestic currency. At $t = 1$, after the complete resolution of the exchange rate uncertainty, the MNF acquires additional k_1 units of capital at a known per-unit price, p_1, in the home country, where p_1 is denominated in the domestic currency and $p_1 > p_0$ to reflect costly expandability of FDI. The MNF's total FDI is thus equal to $k_0 + k_1$.

Given a realization of the spot exchange rate, e, and an initial level of FDI, k_0, the MNF chooses an additional level of FDI, k_1, so as to maximize its domestic currency profit at $t = 2$ under certainty:

$$\max_{k_1} \; ef(k_0 + k_1) - p_0k_0 - p_1k_1 - c \quad \text{s.t.} \quad k_1 \geq 0. \tag{1}$$

The Kuhn-Tucker condition for program (1) is given by

$$ef'[k_0 + k_1(e, k_0)] - p_1 \leq 0, \tag{2}$$

where $k_1(e, k_0)$ is the solution to program (1). If $e \leq p_1/f'(k_0)$, it follows from condition (2) that $k_1(e, k_0) = 0$. On the other hand, if $e > p_1/f'(k_0)$, condition (2) holds as an equality:

$$ef'[k_0 + k_1(e, k_0)] - p_1 = 0, \tag{3}$$

Since $f''(k) < 0$, it is easily verified that $k_1(e, k_0)$ is strictly increasing in e for all $e > p_1/f'(k_0)$. The flexibility of making sequential FDI thus offers the MNF a real (call) option to buy additional capital at $t = 1$, which is exercised whenever $e > p_1/f'(k_0)$.

The MNF is risk averse and possesses a von Neumann-Morgenstern utility function, $u(\pi)$, defined over its domestic currency profit at $t = 2$, π, with $u'(\pi) > 0$ and $u''(\pi) < 0$.[4]

[4]The risk-averse behavior of the MNF can be motivated by managerial risk aversion (Stulz, 1984), corporate taxes (Smith and Stulz, 1985), costs of financial distress (Smith and Stulz, 1985), and capital market imperfections (Froot, Scharfstein, and Stein, 1993; Stulz, 1990). See Tufano (1996) for evidence that managerial risk aversion is a rationale for corporate risk management in the gold mining industry.

To hedge the exchange rate risk at $t = 0$, the MNF has access to a currency forward market wherein the MNF can sell (purchase if negative) h units of the foreign currency forward at a predetermined exchange rate, e^f, expressed in units of the domestic currency per unit of the foreign currency at $t = 1$. To focus on the MNF's hedging motive, vis-à-vis its speculative motive, we assume that the currency forward market is unbiased, i.e., we assume that $e^f = \mathrm{E}(\tilde{e})$, where $\mathrm{E}(\cdot)$ is the expectation operator with respect to $F(e)$.[5]

Figure 1 depicts how the sequence of events unfolds in the model.

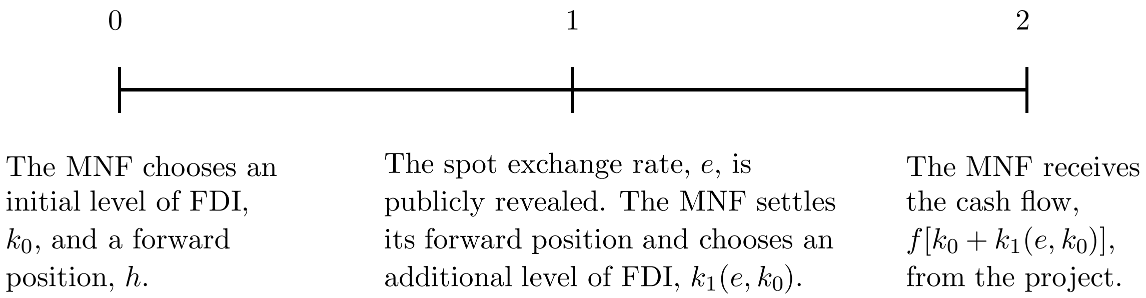

Figure 1. Time Line

The MNF's random domestic currency profit at $t = 2$ is given by

$$\pi(\tilde{e}) = \tilde{e} f[k_0 + k_1(\tilde{e}, k_0)] - p_0 k_0 - p_1 k_1(\tilde{e}, k_0) - c + [\mathrm{E}(\tilde{e}) - \tilde{e}]h, \tag{4}$$

where $[\mathrm{E}(\tilde{e}) - e]h$ is the gain or loss from its forward position, h, and $k_1(e, k_0) = 0$ for all $e \le p_1/f'(k_0)$ and $k_1(e, k_0)$ is defined in Eq. (3) for all $e > p_1/f'(k_0)$. The MNF's ex-ante decision problem is to choose an initial level of FDI, k_0, and a forward position, h, at $t = 0$ so as to maximize the expected utility of its domestic currency profit at $t = 2$:

$$\max_{k_0, h} \; \mathrm{E}\{u[\pi(\tilde{e})]\}, \tag{5}$$

[5]If $e^f >$ ($<$) $\mathrm{E}(\tilde{e})$, the positive (negative) risk premium induces the MNF to speculate in the biased currency forward market by selling (purchasing) the foreign currency forward.

where $\pi(\tilde{e})$ is defined in Eq. (4).

Given the Inada conditions on $f(k)$ and the fact that $p_1 > p_0$, the solution to program (5) must be an interior one.[6] The first-order conditions for program (5) are given by

$$\int_{\underline{e}}^{p_1/f'(k_0^*)} u'[\pi^*(e)][ef'(k_0^*) - p_0]\ \mathrm{d}F(e) + \int_{p_1/f'(k_0^*)}^{\overline{e}} u'[\pi^*(e)](p_1 - p_0)\ \mathrm{d}F(e) = 0, \tag{6}$$

$$\mathrm{E}\{u'[\pi^*(\tilde{e})][\mathrm{E}(\tilde{e}) - \tilde{e}]\} = 0, \tag{7}$$

where Eq. (6) follows from Leibniz's rule and Eq. (3), and an asterisk (*) signifies an optimal level.

3. Lumpy versus sequential FDI

As a benchmark, we consider first a hypothetical case wherein the MNF is unable to adjust its irreversible FDI at $t = 1$ or, equivalently, we set $p_1 = \infty$. In this benchmark case, the MNF's random domestic currency profit at $t = 2$ reduces to

$$\bar{\pi}(\tilde{e}) = \tilde{e}f(k_0) - p_0 k_0 - c + [\mathrm{E}(\tilde{e}) - \tilde{e}]h. \tag{8}$$

At $t = 0$, the MNF chooses a level of FDI, k_0, and a forward position, h, so as to maximize the expected utility of its domestic currency profit at $t = 2$:

$$\max_{k_0, h}\ \mathrm{E}\{u[\bar{\pi}(\tilde{e})]\}, \tag{9}$$

where $\bar{\pi}(\tilde{e})$ is defined in Eq. (8).

The first-order conditions for program (9) are given by

$$\mathrm{E}\{u'[\bar{\pi}^{**}(\tilde{e})][\tilde{e}f'(k_0^{**}) - p_0]\} = 0, \tag{10}$$

[6] If $p_1 = p_0$, it is evident from Eq. (6) that we have a corner solution to program (5) in that $k_0^* = 0$.

$$\mathrm{E}\{u'[\bar{\pi}^{**}(\tilde{e})][\mathrm{E}(\tilde{e}) - \tilde{e}]\} = 0, \tag{11}$$

where a double asterisk (**) indicates an optimal level. Solving Eqs. (10) and (11) yields our first proposition.[7]

Proposition 1. *If the risk-averse MNF is unable to adjust its irreversible FDI at $t = 1$, the MNF's optimal level of lumpy FDI, k_0^{**}, solves*

$$\mathrm{E}(\tilde{e})f'(k_0^{**}) = p_0, \tag{12}$$

and its optimal forward position, h^{**}, satisfies that $h^{**} = f(k_0^{**})$.

To see the intuition of Proposition 1, we recast Eq. (8) as

$$\bar{\pi}(e) = \mathrm{E}(\tilde{e})f(k_0) - p_0k_0 - c + [\mathrm{E}(\tilde{e}) - e][h - f(k_0)]. \tag{13}$$

Inspection of Eq. (13) reveals that the MNF could have completely eliminated its exchange rate risk exposure had it chosen $h = f(k_0)$ within its own discretion. Alternatively put, the degree of exchange rate risk exposure to be assumed by the MNF should be totally unrelated to its FDI decision at $t = 0$. The optimal level of lumpy FDI, k_0^{**}, is then chosen to maximize $\mathrm{E}(\tilde{e})f(k_0) - p_0k_0$, which yields Eq. (12). Since the currency forward market is unbiased, it offers actuarially fair "insurance" to the MNF. The risk-averse MNF as such optimally opts for full insurance by choosing $h^{**} = f(k_0^{**})$, which completely eliminates its exchange rate risk exposure. These results are simply the well-known separation and full-hedging theorems emanated from the literature on MNFs under exchange rate uncertainty (see, e.g., Adam-Müller, 1997; Broll, 1992; Broll and Zilcha, 1992; Chang and Wong, 2003; Lien and Wang, 2006; Wong, 2003a, 2003b).

Proposition 2. *If the risk-averse MNF is allowed to make sequential FDI, the MNF's optimal initial level of FDI, k_0^*, is less than the optimal level of lumpy FDI, k_0^{**}, and its optimal forward position, h^*, satisfies that $h^* > f(k_0^*)$.*

[7] All proofs of propositions are given in Appendix A.

The intuition of Proposition 2 is as follows. The flexibility of making sequential FDI, vis-à-vis lumpy FDI, offers the MNF a real (call) option to buy additional capital at $t = 1$, which is exercised whenever $e > p_1/f'(k_0)$. It is well-known that the value of a call option increases with a decrease in its exercise price (Merton, 1973). Since $f''(k) < 0$, the MNF has incentives to cut down its initial level of FDI, k_0, so as to lower $p_1/f'(k_0)$, the exercise price of the real option created by the flexibility of making sequential FDI. Thus, we have $k_0^* < k_0^{**}$. Given the fact that its exchange rate risk exposure is at least $f(k_0^*)$ and is strictly greater than $f(k_0^*)$ when $e > p_1/f'(k_0^*)$, the MNF opts for $h^* > f(k_0^*)$ as its optimal forward position. Unlike in the case of lumpy FDI, the MNF has to bear some residual exchange rate risk that cannot be eliminated by currency forward hedging.

Using a sample of U.S. MNFs, Allayannis, Ihrig, and Weston (2001) find that operational hedging is not an effective substitute for financial hedging. In spite of this, operational hedging is capable of reducing exchange rate risk exposure and enhancing firm value when it is used in combination with financial hedging. This is confirmed by Pantzalis, Simkins, and Laux (2001) and Kim, Mathur, and Nam (2006) who also show that operational hedging serves as a real option for managing exchange rate risk. Furthermore, Allayannis, Ihrig, and Weston (2001) and Kim, Mathur, and Nam (2006) find that operational hedging is a complement to, not a substitute for, financial hedging. From Proposition 1, we know that a full-hedge is an optimal forward position in the case of lumpy FDI. The over-hedging result of Proposition 2 when the MNF is allowed to make sequential FDI is thus consistent with the empirical finding that operational hedges and financial hedges are complements to each other.

Proposition 2 states that the MNF is forced to undertake more FDI at $t = 0$ should FDI be lumpy than sequential. It is of interest to extend this result by comparing the expected optimal aggregate level of sequential FDI, $k_0^* + \mathrm{E}[k_1(\tilde{e}, k_0^*)]$, with that of lumpy FDI, k_0^{**}. Since $k_0^* < k_0^{**}$ and $\mathrm{E}[k_1(\tilde{e}, k_0^*)] > 0$, such a comparison is a non-trivial one. To see this, consider the extreme case wherein $p_1 = p_0$. In this case, we know from Eq. (6) that $k_0^* = 0$. It then follows from Eqs. (3) and (12) that $k_1[\mathrm{E}(\tilde{e}), 0] = k_0^{**}$. Thus, to show $\mathrm{E}[k_1(\tilde{e}, 0)] > (<)\ k_0^{**}$, it suffices to show that $k_1(e, 0)$ is convex (concave) in e according to

Jensen's inequality.

Proposition 3. *Given that $p_1 = p_0$, the expected optimal level of sequential FDI,* $\mathrm{E}[k_1(\tilde{e}, 0)]$, *is greater or smaller than the optimal level of lumpy FDI, k_0^{**}, depending on whether $f'(k)f'''(k)/f''(k)^2$ is everywhere no less or no greater than 2, respectively.*

If $f'''(k) \leq 0$, then $f'(k)f'''(k)/f''(k)^2 \leq 0$ so that $k_1(e, 0)$ is concave in e. In this case, we have $\mathrm{E}[k_1(\tilde{e}, 0)] < k_0^{**}$. On the other hand, if $f(k) = k^\alpha$, where $0 < \alpha < 1$, then $f'(k)f'''(k)/f''(k)^2 = 2 + \alpha/(1 - \alpha) > 2$ so that $k_1(e, 0)$ is convex in e. In this case, we have $\mathrm{E}[k_1(\tilde{e}, 0)] > k_0^{**}$. In general, without knowing the specific functional forms of $u(\pi)$, $f(k)$, and $F(e)$, we are a priori unable to make an unambiguous comparison between the expected optimal aggregate level of sequential FDI and that of lumpy FDI.

4. Sequential FDI without forward hedging

In this section, we restrict our attention to the case wherein the MNF is banned from trading in the currency forward market. This is tantamount to setting $h \equiv 0$. In this case, the MNF's random domestic currency profit at $t = 2$ reduces to

$$\hat{\pi}(\tilde{e}) = \tilde{e}f[k_0 + k_1(\tilde{e}, k_0)] - p_0 k_0 - p_1 k_1(\tilde{e}, k_0) - c. \tag{14}$$

At $t = 0$, the MNF chooses a level of FDI, k_0, so as to maximize the expected utility of its domestic currency profit at $t = 2$:

$$\max_{k_0} \ \mathrm{E}\{u[\hat{\pi}(\tilde{e})]\}, \tag{15}$$

where $\hat{\pi}(\tilde{e})$ is defined in Eq. (14).

The first-order condition for program (15) is given by

$$\int_{\underline{e}}^{p_1/f'(k_0^\diamond)} u'[\hat{\pi}^\diamond(e)][ef'(k_0^\diamond) - p_0] \,\mathrm{d}F(e) + \int_{p_1/f'(k_0^\diamond)}^{\overline{e}} u'[\hat{\pi}^\diamond(e)](p_1 - p_0) \,\mathrm{d}F(e) = 0, \tag{16}$$

where a diamond ($^\diamond$) indicates an optimal level. Rearranging terms of Eq. (16) yields

$$\mathrm{E}\left\{\frac{u'[\hat{\pi}^\diamond(\tilde{e})]}{\mathrm{E}\{u'[\hat{\pi}^\diamond(\tilde{e})]\}}\tilde{e}\right\}f'(k_0^\diamond) = p_0 + \mathrm{E}\left\{\frac{u'[\hat{\pi}^\diamond(\tilde{e})]}{\mathrm{E}\{u'[\hat{\pi}^\diamond(\tilde{e})]\}}\max[\tilde{e}f'(k_0^\diamond) - p_1, 0]\right\}. \tag{17}$$

Define the following function:

$$G(e) = \int_{\underline{e}}^{e} \frac{u'[\hat{\pi}^\diamond(x)]}{\mathrm{E}\{u'[\hat{\pi}^\diamond(\tilde{e})]\}}\,\mathrm{d}F(x), \tag{18}$$

for all $e \in [\underline{e},\ \overline{e}]$. It is evident from Eq. (18) that $G'(e) > 0$, $G(\underline{e}) = 0$, and $G(\overline{e}) = 1$. We can as such interpret $G(e)$ as a cumulative distribution function of $\tilde{e}$. Substituting Eq. (18) into Eq. (17) yields

$$\mathrm{E}_G(\tilde{e})f'(k_0^\diamond) = p_0 + \mathrm{E}_G\{\max[\tilde{e}f'(k_0^\diamond) - p_1, 0]\}, \tag{19}$$

where $\mathrm{E}_G(\cdot)$ is the expectation operator with respect to $G(e)$. Eq. (19) states that the optimal initial level of FDI is the one that equates the expected marginal return to FDI made at $t = 0$ to the per-unit price of capital at $t = 0$ plus the forgone option value of waiting to invest that unit of capital at $t = 1$, where the expectations are evaluated taking the MNF's risk attitude into account.

If the MNF is risk neutral so that $u(\pi) = \pi$, the optimal initial level of FDI, k_0^c, is given by

$$\mathrm{E}(\tilde{e})f'(k_0^c) = p_0 + \mathrm{E}\{\max[\tilde{e}f'(k_0^c) - p_1, 0]\}, \tag{20}$$

which follows from Eq. (17) by setting $u'(\pi) = 1$. Using the covariance operator with respect to $F(e)$, $\mathrm{Cov}(\cdot,\cdot)$, we can write Eq. (17) as[8]

$$\left\{\mathrm{E}(\tilde{e}) + \frac{\mathrm{Cov}\{u'[\hat{\pi}^\diamond(\tilde{e})], \tilde{e}\}}{\mathrm{E}\{u'[\hat{\pi}^\diamond(\tilde{e})]\}}\right\}f'(k_0^\diamond)$$

$$= p_0 + \left\{\mathrm{E}\{\max[\tilde{e}f'(k_0^\diamond) - p_1, 0]\} + \frac{\mathrm{Cov}\{u'[\hat{\pi}^\diamond(\tilde{e})], \max[\tilde{e}f'(k_0^\diamond) - p_1, 0]\}}{\mathrm{E}\{u'[\hat{\pi}^\diamond(\tilde{e})]\}}\right\}. \tag{21}$$

[8]For any two random variables, $\tilde{x}$ and $\tilde{y}$, we have $\mathrm{Cov}(\tilde{x},\tilde{y}) = \mathrm{E}(\tilde{x}\tilde{y}) - \mathrm{E}(\tilde{x})\mathrm{E}(\tilde{y})$.

From Eqs. (3) and (14), we know that $\hat{\pi}^{\diamond\prime}(e) = f[k_0^\diamond + k_1(e, k_0^\diamond)] > 0$. Since $u''(\pi) < 0$, we have $\mathrm{Cov}\{u'[\hat{\pi}^\diamond(\tilde{e})], \tilde{e}\} < 0$ and $\mathrm{Cov}\{u'[\hat{\pi}^\diamond(\tilde{e})], \max[\tilde{e}f'(k_0^\diamond) - p_1, 0]\} < 0$. Inspection of Eqs. (19), (20), and (21) reveals that risk aversion reduces both the expected marginal return to FDI made at $t = 0$ and the forgone option value of waiting to invest that unit of capital at $t = 1$, as compared to those under risk neutrality. The former has a negative effect on the risk-averse MNF's initial level of FDI while the latter has a positive effect. Using the fact that $\tilde{e}f'(k_0^\diamond) - p_1 = \max[\tilde{e}f'(k_0^\diamond) - p_1, 0] - \max[p_1 - \tilde{e}f'(k_0^\diamond), 0]$, we can write Eq. (21) as

$$\mathrm{E}(\tilde{e})f'(k_0^\diamond) = p_0 + \mathrm{E}\{\max[\tilde{e}f'(k_0^\diamond) - p_1, 0]\}$$

$$+\frac{\mathrm{Cov}\{u'[\hat{\pi}^\diamond(\tilde{e})], \max[p_1 - \tilde{e}f'(k_0^\diamond), 0]\}}{\mathrm{E}\{u'[\hat{\pi}^\diamond(\tilde{e})]\}}, \tag{22}$$

where the net effect is governed by the covariance term on the right-hand side of Eq. (22) and is unambiguously negative (since the covariance term is unambiguously positive).

Proposition 4. *Suppose that the MNF is initially risk neutral and is banned from trading in the currency forward market. Introducing risk aversion reduces the MNF's ex-ante and ex-post incentives to make FDI in that* $k_0^\diamond < k_0^c$ *and* $k_0^\diamond + k_1(e, k_0^\diamond) \leq k_0^c + k_1(e, k_0^c)$*, where the inequality is strict for all* $e < p_1/f'(k_0^c)$*.*

To see the intuition underlying Proposition 4, we partially differentiate Eq. (14) with respect to k_0 to yield

$$\frac{\partial \hat{\pi}(e)}{\partial k_0} = ef'[k_0 + k_1(e, k_0)] - p_0, \tag{23}$$

which is negative for all $e < p_0/f'(k_0)$ and positive for all $e > p_0/f'(k_0)$. If the MNF is risk neutral, the optimal initial level of FDI is k_0^c. When the MNF becomes risk averse, it has incentives to shift its domestic currency profits when the realizations of $\tilde{e}$ are high to those when the realizations of $\tilde{e}$ are low. This can be done by lowering k_0, as is evident from Eq. (23). Hence, we must have $k_0^\diamond < k_0^c$. It follows from Eq. (3) that $k_0^\diamond + k_1(e, k_0^\diamond) < k_0^c$ for all $e < p_1/f'(k_0^c)$ and $k_0^\diamond + k_1(e, k_0^\diamond) = k_0^c + k_1(e, k_0^c)$ for all $e \geq p_1/f'(k_0^c)$. Thus, both the optimal initial level of FDI and the optimal aggregate level of FDI are lower when the MNF is risk averse than when it is risk neutral.

Proposition 4 shows a perverse effect of a global increase in risk aversion on the MNF's ex-ante and ex-post incentives to made FDI. It is of interest to verify that such a negative effect on FDI is preserved in the case of a marginal increase in risk aversion. To this end, let $v(\pi)$ be a von Neumann-Morgenstern utility function that is more risk averse than $u(\pi)$. According to Pratt (1964), we can write $v(\pi) = \phi[u(\pi)]$, where $\phi(\cdot)$ is a strictly concave function. The more risk-averse MNF's ex-ante decision problem is given by

$$\max_{k_0} \ \mathrm{E}\{v[\hat{\pi}(\tilde{e})]\}, \tag{24}$$

where $\hat{\pi}(\tilde{e})$ is defined in Eq. (14).

Proposition 5. *Suppose that the risk-averse MNF is banned from trading in the currency forward market. The perverse effect of a global increase in risk aversion on the MNF's ex-ante and ex-post incentives to make FDI, as characterized in Proposition 4, is preserved in the case of a marginal increase in risk aversion from $u(\pi)$ to $v(\pi)$.*

The intuition of Proposition 5 is the same as that of Proposition 4 and thus is omitted.

Finally, we want to examine how the fixed cost, c, for the access to the project would affect the MNF's incentives to make FDI. As is well known in the literature on decision making under certainty, risk aversion alone is usually too weak to yield intuitively appealing comparative statics. To reconcile these shortcomings, the literature suggests that it is reasonable and useful to impose the additional assumption of decreasing absolute risk aversion (see Gollier, 2001). We say that the MNF's utility function, $u(\pi)$, exhibits decreasing absolute risk aversion if, and only if, its Arrow-Pratt measure of absolute risk aversion, $-u''(\pi)/u'(\pi)$, decreases with π.[9]

[9] If the MNF's utility function satisfies increasing (constant) absolute risk aversion, it can be shown analogously that $\mathrm{d}k_0^{\diamond}/\mathrm{d}c > (=)\ 0$.

Proposition 6. *Suppose that the risk-averse MNF is banned from trading in the currency forward market. If the MNF's utility function exhibits decreasing absolute risk aversion, an increase in the fixed cost for the access to the project reduces the MNF's ex-ante and ex-post incentives to make FDI.*

The intuition of Proposition 6 is as follows. An increase in the fixed cost, c, reduces the MNF's domestic currency profit by the same amount for all $e \in [\underline{e}, \overline{e}]$. Given decreasing absolute risk aversion, the MNF becomes more risk averse. It then follows from Proposition 5 that the MNF's ex-ante and ex-post incentives to make FDI are reduced.

Changes in fixed or setup costs incurred by MNFs may be due to changes in investment tax credit offered by the host government, or due to changes in the severity of entry barriers in the host country. Proposition 6 thus implies that FDI flows are positively related to higher investment tax credits and negatively related to more barriers to entry. Hines (2001) finds that the volume of Japanese FDI in countries with whom Japan has "tax sparing" agreements is 1.4 to 2.4 times higher than it would have been otherwise. Furthermore, Anand and Kogut (1997) document that industrial concentration has a negative effect on the attraction of FDI. The implications of Proposition 6 are largely consistent with these empirical findings.

5. Sequential FDI with forward hedging

In this section, we resume the MNF's access to the currency forward market. Rearranging terms of Eq. (6) yields

$$\mathrm{E}\{u'[\pi^*(\tilde{e})][\tilde{e}f'(k_0^*) - p_0]\} - \mathrm{E}\{u'[\pi^*(\tilde{e})]\max[\tilde{e}f'(k_0^*) - p_1, 0]\} = 0. \tag{25}$$

Multiplying $f'(k_0^*)$ to Eq. (7) and adding the resulting equation to Eq. (25) yields

$$\mathrm{E}(\tilde{e})f'(k_0^*) = p_0 + \mathrm{E}\left\{\frac{u'[\pi^*(\tilde{e})]}{\mathrm{E}\{u'[\pi^*(\tilde{e})]\}}\max[\tilde{e}f'(k_0^*) - p_1, 0]\right\}. \tag{26}$$

Define the following function:

$$H(e) = \int_{\underline{e}}^{e} \frac{u'[\pi^*(x)]}{\mathrm{E}\{u'[\pi^*(\tilde{e})]\}}\, \mathrm{d}F(x), \tag{27}$$

for all $e \in [\underline{e},\ \overline{e}]$. It is evident from Eq. (27) that $H'(e) > 0$, $H(\underline{e}) = 0$, and $H(\overline{e}) = 1$. We can as such interpret $H(e)$ as a cumulative distribution function of $\tilde{e}$. Substituting Eq. (27) into Eq. (26) yields

$$\mathrm{E}(\tilde{e})f'(k_0^*) = p_0 + \mathrm{E}_H\{\max[\tilde{e}f'(k_0^*) - p_1, 0]\}, \tag{28}$$

where $\mathrm{E}_H(\cdot)$ is the expectation operator with respect to $H(e)$.

The interpretation of Eq. (28) is similar to those of Eqs. (19) and (20) with two caveats. First, when the currency forward market is available for hedging purposes, the MNF's risk preferences play no role in determining the expected return to FDI made at $t = 0$, which is governed solely by $F(e)$, as is evident from the left-hand side of Eq. (28). This is simply the spanning property that arises from the tradability of $\tilde{e}$ in the currency forward market. Second, the option value of waiting to make FDI at $t = 1$ is now priced based on $H(e)$, as is evident from the right-hand side of Eq. (28). Using the covariance operator with respect to $F(e)$, we can write this option value as

$$\begin{aligned}\mathrm{E}_H\{\max[\tilde{e}f'(k_0^*) - p_1, 0]\} = \mathrm{E}\{\max[\tilde{e}f'(k_0^*) - p_1, 0]\} \\ + \frac{\mathrm{Cov}\{u'[\pi^*(\tilde{e})], \max[\tilde{e}f'(k_0^*) - p_1, 0]\}}{\mathrm{E}\{u'[\pi^*(\tilde{e})]\}}.\end{aligned} \tag{29}$$

The wedge between this option value and the option value under risk neutrality is gauged by the covariance term on the right-hand side of Eq. (29). Due to the non-tradability of the real option embedded in sequential FDI, spanning is not possible and thus the MNF's risk preferences affect the pricing of the option in this incomplete market context.

Partially differentiating $\pi^*(e)$ with respect to e yields

$$\pi^{*'}(e) = f[k_0^* + k_1(e, k_0^*)] - h^*, \tag{30}$$

where we have used Eq. (3). From Proposition 2, we know that $h^* > f(k_0^*)$. Eq. (30) then implies that $\pi^*(e)$ is strictly decreasing for all $e < e_0$ and strictly increasing for all $e > e_0$, where e_0 solves $f[k_0^* + k_1(e_0, k_0^*)] = h^*$. Since $\pi^*(e)$ is non-monotonic in e, the sign of the covariance term on the right-hand side of Eq. (29) is not immediately determinate.

Proposition 7. *Suppose that the MNF is initially risk neutral and has access to the currency forward market. Introducing risk aversion improves the MNF's ex-ante and ex-post incentives to make FDI in that $k_0^* > k_0^c$ and $k_0^* + k_1(e, k_0^*) \geq k_0^c + k_1(e, k_0^c)$, where the inequality is strict for all $e < p_1/f'(k_0^*)$.*

The results of Proposition 7 should be contrasted with those of Proposition 4. In the presence of the currency forward market, risk aversion has no effect on the expected marginal return to FDI made at $t = 0$ but has a negative effect on the forgone option value of waiting to invest that unit of capital at $t = 1$, as is evident from Eq. (28). Thus, the MNF is induced to make more FDI at $t = 0$ so that $k_0^* > k_0^c$. It then follows from Eq. (3) that $k_0^* > k_0^c + k_1(e, k_0^c)$ for all $e < p_1/f'(k_0^*)$ and $k_0^* + k_1(e, k_0^*) = k_0^c + k_1(e, k_0^c)$ for all $e \geq p_1/f'(k_0^*)$. Thus, both the optimal initial level of FDI and the optimal aggregate level of FDI are higher when the MNF is risk averse than when it is risk neutral. The opposite results, however, hold when the currency forward market is not available to the MNF for hedging purposes (see Proposition 4). An immediate implication is that forward hedging promotes FDI, both ex ante and ex post, a result in line with the extant literature on lumpy FDI (Adam-Müller, 1997; Broll, 1992; Broll and Zilcha, 1992; Broll, Wong, and Zilcha, 1999; Wong, 2003b). This is consistent with the complementary nature of operational and financial hedging strategies as empirically documented by Allayannis, Ihrig, and Weston (2001) and Kim, Mathur, and Nam (2006).

6. Conclusions

This paper has examined the behavior of a risk-averse multinational firm (MNF) that has an investment opportunity in a foreign country. The MNF makes sequential foreign direct investment (FDI) in response to the intertemporal resolution of exchange rate uncertainty. To hedge the ex-ante exchange rate risk, the MNF has access to an unbiased currency forward market.

Within this framework, we have shown that neither the separation theorem nor the full-hedging theorem holds. The flexibility of making sequential FDI, vis-à-vis lumpy FDI, offers the MNF a real (call) option that is rationally exercised whenever the foreign currency has been substantially appreciated relative to the domestic currency. We have shown that the MNF's optimal initial level of sequential FDI is always lower than that of lumpy FDI, while the expected optimal aggregate level of sequential FDI can be higher or lower than that of lumpy FDI. Consistent with the observed hedging behavior of non-financial firms in the U.S. (Bodnar, Hayt, and Marston, 1998), the MNF does not fully hedge its exchange rate risk exposure.

In the absence of the currency forward market, we have show that the MNF's ex-ante and ex-post incentives to make FDI are reduced with either a global or a marginal increase in risk aversion. An increase in the fixed or setup cost incurred by the MNF generates the same perverse effect on FDI if the MNF's utility function satisfies the reasonable property of decreasing absolute risk aversion. Given that the change in the fixed or setup cost may be due to a change in the investment tax credit offered by the host government, or due to a change in the severity of entry barriers in the host country, FDI flows are expected to react in a predictable manner when these government policies and market conditions shift over time. Finally, we have shown that the MNF's ex-ante and ex-post incentives to make FDI are improved when the MNF's access to the currency forward market is resumed. This implies immediately that forward hedging promotes FDI, a result in line with the extant literature on lumpy FDI (Adam-Müller, 1997; Broll, 1992; Broll and Zilcha, 1992; Broll, Wong, and Zilcha, 1999; Wong, 2003b).

Appendix A

Proof of Proposition 1. Multiplying $f'(k_0^{**})$ to Eq. (11) and adding the resulting equation to Eq. (10) yields

$$\mathrm{E}\{u'[\bar{\pi}^{**}(\tilde{e})][\mathrm{E}(\tilde{e})f'(k_0^{**}) - p_0]\} = 0. \tag{A.1}$$

Since $u'(\pi) > 0$, Eq. (A.1) reduces to Eq. (12).

Using the covariance operator with respect to $F(e)$, we can write Eq. (11) as

$$\mathrm{Cov}\{u'[\bar{\pi}^{**}(\tilde{e})], \tilde{e}]\} = 0. \tag{A.2}$$

If $h^{**} = f(k_0^{**})$, it follows from Eq. (8) that $\bar{\pi}^{**}(e) = \mathrm{E}(\tilde{e})f(k_0^{**}) - p_0 k_0^{**}$, which is invariant to e. Hence, $h^{**} = f(k_0^{**})$ solves Eq. (A.2). □

Proof of Proposition 2. Rearranging terms of Eq. (6) yields

$$\mathrm{E}\{u'[\pi^*(\tilde{e})][\tilde{e}f'(k_0^*) - p_0]\} - \mathrm{E}\{u'[\pi^*(\tilde{e})]\max[\tilde{e}f'(k_0^*) - p_1, 0]\} = 0. \tag{A.3}$$

Multiplying $f'(k_0^*)$ to Eq. (7) and adding the resulting equation to Eq. (A.3) yields

$$\mathrm{E}(\tilde{e})f'(k_0^*) = p_0 + \mathrm{E}\left\{\frac{u'[\pi^*(\tilde{e})]}{\mathrm{E}\{u'[\pi^*(\tilde{e})]\}}\max[\tilde{e}f'(k_0^*) - p_1, 0]\right\} > p_0, \tag{A.4}$$

where the inequality follows from $u'(\pi) > 0$ and $\max[\tilde{e}f'(k_0^*) - p_1, 0] \geq 0$. Since $f''(k) < 0$, Eqs. (12) and (A.4) imply that $k_0^* < k_0^{**}$.

Using the covariance operator with respect to $F(e)$, we can write Eq. (7) as

$$\mathrm{Cov}\{u'[\pi^*(\tilde{e})], \tilde{e}\} = 0. \tag{A.5}$$

Partially differentiating $u'[\pi^*(e)]$ with respect to e yields

$$\frac{\partial}{\partial e}u'[\pi^*(e)] = u''[\pi^*(e)]\{f[k_0^* + k_1(e, k_0^*)] - h^*\}, \tag{A.6}$$

where we have used Eq. (3). Suppose that $h^* \leq f(k_0^*)$. Since $k_1(e, k_0^*) \geq 0$ and $f'(k) > 0$, Eq. (A.6) implies that $\mathrm{Cov}\{u'[\pi^*(\tilde{e})], \tilde{e}\} < 0$, which contradicts Eq. (A.5). Thus, it must be true that $h^* > f(k_0^*)$. □

Proof of Proposition 3. Totally differentiating $ef'[k_1(e, 0)] = p_0$ with respect to e twice and rearranging terms yields

$$\frac{\partial^2}{\partial e^2} k_1(e, 0) = -\frac{f'[k_1(e, 0)]}{e^2 f''[k_1(e, 0)]}\left\{\frac{f'[k_1(e, 0)]f'''[k_1(e, 0)]}{f''[k_1(e, 0)]^2} - 2\right\}. \tag{A.7}$$

It follows from Eq. (A.7) that $k_1(e, 0)$ is convex (concave) in e if $f'(k)f'''(k)/f''(k)^2$ is everywhere no less (no greater) than 2. By Jensen's inequality, the convexity (concavity) of $k_1(e, 0)$ in e implies that $\mathrm{E}[k_1(\tilde{e}, 0)] > (<)\ k_1[\mathrm{E}(\tilde{e}), 0] = k_0^{**}$. □

Proof of Proposition 4. Since $\mathrm{Cov}\{u'[\hat{\pi}^\diamond(\tilde{e})], \max[p_1 - \tilde{e}f'(k_0^\diamond), 0]\} > 0$, Eq. (22) implies that $\mathrm{E}(\tilde{e})f'(k_0^\diamond) > p_0 + \mathrm{E}\{\max[\tilde{e}f'(k_0^\diamond) - p_1, 0]\}$. It then follows from Eq. (20) and the strict concavity of $f(k)$ that $k_0^\diamond < k_0^c$. From Eq. (3), we have $k_0^\diamond + k_1(e, k_0^\diamond) < k_0^c$ for all $e < p_1/f'(k_0^c)$ and $k_0^\diamond + k_1(e, k_0^\diamond) = k_0^c + k_1(e, k_0^c)$ for all $e \geq p_1/f'(k_0^c)$. □

Proof of Proposition 5. Differentiating the objective function in program (24) with respect to k_0, and evaluating the resulting derivative at $k_0 = k_0^\diamond$ yields

$$\left.\frac{\mathrm{dE}\{v[\hat{\pi}(\tilde{e})]\}}{\mathrm{d}k_0}\right|_{k_0 = k_0^\diamond} = \int_{\underline{e}}^{p_1/f'(k_0^\diamond)} \phi'\{u[\hat{\pi}^\diamond(e)]\}u'[\hat{\pi}^\diamond(e)][ef'(k_0^\diamond) - p_0]\ \mathrm{d}F(e)$$

$$+ \int_{p_1/f'(k_0^\diamond)}^{\overline{e}} \phi'\{u[\hat{\pi}^\diamond(e)]\}u'[\hat{\pi}^\diamond(e)](p_1 - p_0)\ \mathrm{d}F(e). \tag{A.8}$$

Multiplying $\phi'\{u\{\hat{\pi}^\diamond[p_0/f'(k_0^\diamond)]\}\}$ to Eq. (16) and substituting the resulting equation to the right-hand side of Eq. (A.8) yields

$$\int_{\underline{e}}^{p_1/f'(k_0^\diamond)} \Big\{\phi'\{u[\hat{\pi}^\diamond(e)]\} - \phi'\{u\{\hat{\pi}^\diamond[p_0/f'(k_0^\diamond)]\}\}\Big\}u'[\hat{\pi}^\diamond(e)][ef'(k_0^\diamond) - p_0]\ \mathrm{d}F(e)$$

$$+ \int_{p_1/f'(k_0^\diamond)}^{\overline{e}} \Big\{\phi'\{u[\hat{\pi}^\diamond(e)]\} - \phi'\{u\{\hat{\pi}^\diamond[p_0/f'(k_0^\diamond)]\}\}\Big\}u'[\hat{\pi}^\diamond(e)](p_1 - p_0)\ \mathrm{d}F(e).$$

The above expression is unambiguously negative because the strict concavity of $\phi(\cdot)$ implies that $\phi'\{u[\hat{\pi}^\diamond(e)]\} > (<)\ \phi'\{u\{\hat{\pi}^\diamond[p_0/f'(k_0^\diamond)]\}\}$ for all $e < (>)\ p_0/f'(k_0^\diamond)$. Thus, the MNF must invest less than $k_0^\diamond$ at $t = 0$ when it becomes more risk averse. □

Proof of Proposition 6. Totally differentiating Eq. (16) with respect to c and rearranging terms yields

$$\frac{\mathrm{d}k_0^\diamond}{\mathrm{d}c} = \frac{1}{\Delta}\bigg\{\int_{\underline{e}}^{p_1/f'(k_0^\diamond)} u''[\hat{\pi}^\diamond(e)][ef'(k_0^\diamond) - p_0]\ \mathrm{d}F(e) + \int_{p_1/f'(k_0^\diamond)}^{\overline{e}} u''[\hat{\pi}^\diamond(e)](p_1 - p_0)\ \mathrm{d}F(e)\bigg\}, \tag{A.9}$$

where $\Delta = \int_{\underline{e}}^{p_1/f'(k_0^\diamond)} u''[\hat{\pi}^\diamond(e)][ef'(k_0^\diamond) - p_0]^2\ \mathrm{d}F(e) + \int_{\underline{e}}^{p_1/f'(k_0^\diamond)} u'[\hat{\pi}^\diamond(e)]ef''(k_0^\diamond)\ \mathrm{d}F(e) + \int_{p_1/f'(k_0^\diamond)}^{\overline{e}} u''[\hat{\pi}^\diamond(e)](p_1 - p_0)^2\ \mathrm{d}F(e) < 0$. From Eqs. (3) and (14), we have $\hat{\pi}^{\diamond\prime}(e) = f[k_0^\diamond + k_1(e, k_0^\diamond)] > 0$. Since $u(\pi)$ satisfies decreasing absolute risk aversion, we have

$$-\frac{u''[\hat{\pi}^\diamond(e)]}{u'[\hat{\pi}^\diamond(e)]} > (<)\ R \quad \text{for all} \quad e < (>)\ p_0/f'(k_0^\diamond), \tag{A.10}$$

where R is the Arrow-Pratt measure of absolute risk aversion evaluated at $e = p_0/f'(k_0^\diamond)$. We multiply $-u'[\hat{\pi}^\diamond(e)][ef'(k_0^\diamond) - p_0]$ to both sides of inequality (A.10) for all $e < p_0/f'(k_0^\diamond)$, and $-u'[\hat{\pi}^\diamond(e)](p_1 - p_0)$ to both sides of inequality (A.10) for all $e > p_0/f'(k_0^\diamond)$. Taking the expectations on both sides of the resulting inequality with respect to $F(e)$ yields

$$\begin{aligned}
&\int_{\underline{e}}^{p_1/f'(k_0^\diamond)} u''[\hat{\pi}^\diamond(e)][ef'(k_0^\diamond) - p_0]\ \mathrm{d}F(e) + \int_{p_1/f'(k_0^\diamond)}^{\overline{e}} u''[\hat{\pi}^\diamond(e)](p_1 - p_0)\ \mathrm{d}F(e) \\
&> -R\bigg\{\int_{\underline{e}}^{p_1/f'(k_0^\diamond)} u'[\hat{\pi}^\diamond(e)][ef'(k_0^\diamond) - p_0]\ \mathrm{d}F(e) + \int_{p_1/f'(k_0^\diamond)}^{\overline{e}} u'[\hat{\pi}^\diamond(e)](p_1 - p_0)\ \mathrm{d}F(e)\bigg\} \\
&= 0,
\end{aligned} \tag{A.11}$$

where the equality follows from Eq. (16). Hence, Eq. (A.9) and inequality (A.11) imply that $\mathrm{d}k_0^\diamond/\mathrm{d}c < 0$. □

Proof of Proposition 7. From Proposition 2, we know that $h^* > f(k_0^*)$. Eq. (A.6) then implies that $u'[\pi^*(e)]$ is strictly increasing for all $e < e_0$ and strictly decreasing for all $e > e_0$, where e_0 solves $f[k_0^*, k_1(e_0, k_1^*)] = h^*$. In other words, $u'[\pi^*(e)]$ is hump-shaped and attains a unique global maximum at $e = e_0$. Since $\mathrm{E}\{u'[\pi^*(\tilde{e})]\}$ is the expected value of $u'[\pi^*(\tilde{e})]$, there must exist at least one and at most two distinct points at which $u'[\pi^*(e)] = \mathrm{E}\{u'[\pi^*(\tilde{e})]\}$. Write Eq. (7) as

$$\int_{\underline{e}}^{\overline{e}} \Big\{u'[\pi^*(e)] - \mathrm{E}\{u'[\pi^*(\tilde{e})]\}\Big\}(e - y)\ \mathrm{d}F(e) = 0, \tag{A.12}$$

for all $y \in [\underline{e}, \overline{e}]$. If there is only one point, $\hat{e}$, at which $u'[\pi^*(\hat{e})] = \mathrm{E}\{u'[\pi^*(\tilde{e})]\}$, then we have

$$\int_{\underline{e}}^{\overline{e}} \Big\{u'[\pi^*(e)] - \mathrm{E}\{u'[\pi^*(\tilde{e})]\}\Big\}(e - \hat{e})\ \mathrm{d}F(e) > (<)\ 0, \tag{A.13}$$

when $u'[\pi^*(\underline{e})] \le (>)\ \mathrm{E}\{u'[\pi^*(\tilde{e})]\}$, a contradiction to Eq. (A.12). Thus, there must exist two distinct points, e_1 and e_2, with $\underline{e} < e_1 < e_0 < e_2 < \overline{e}$, such that $u'[\pi^*(e)] \ge \mathrm{E}\{u'[\pi^*(\tilde{e})]\}$ for all $e \in [e_1, e_2]$ and $u'[\pi^*(e)] < \mathrm{E}\{u'[\pi^*(\tilde{e})]\}$ for all $e \in [\underline{e}, e_1) \bigcup (e_2, \overline{e}]$, where the equality holds only at $e = e_1$ and $e = e_2$.

Consider the following function:

$$\begin{aligned} g(x) &= \mathrm{Cov}\{u'[\pi^*(\tilde{e})], \max[\tilde{e} - x, 0]\} \\ &= \int_{x}^{\overline{e}} \Big\{u'[\pi^*(e)] - \mathrm{E}\{u'[\pi^*(\tilde{e})]\}\Big\}(e - x)\ \mathrm{d}F(e). \end{aligned} \tag{A.14}$$

Differentiating Eq. (A.14) with respect to x and using Leibniz's rule yields

$$g'(x) = -\int_{x}^{\overline{e}} \Big\{u'[\pi^*(e)] - \mathrm{E}\{u'[\pi^*(\tilde{e})]\}\Big\}\ \mathrm{d}F(e). \tag{A.15}$$

Differentiating Eq. (A.15) with respect to x and using Leibniz's rule yields

$$g''(x) = \Big\{u'[\pi^*(x)] - \mathrm{E}\{u'[\pi^*(\tilde{e})]\}\Big\}F'(x). \tag{A.16}$$

It follows from Eq. (A.16) that $g''(x) \geq 0$ for all $x \in [e_1, e_2]$ and $g''(x) < 0$ for all $x \in [\underline{e}, e_1) \bigcup (e_2, \overline{e}]$, where the equality holds only at $x = e_1$ and $x = e_2$. In words, $g(x)$ is strictly concave for all $x \in [\underline{e}, e_1) \bigcup (e_2, \overline{e}]$ and is strictly convex for all $x \in (e_1, e_2)$. It follows from Eq. (A.15) that $g'(\underline{e}) = g'(\overline{e}) = 0$. Hence, $g(x)$ attains two local maxima at $x = \underline{e}$ and $x = \overline{e}$. From Eq. (A.14), we have $g(\overline{e}) = 0$. Also, Eqs. (A.12) and (A.14) imply that

$$g(\underline{e}) = \int_{\underline{e}}^{\overline{e}} \Big\{u'[\pi^*(e)] - \mathrm{E}\{u'[\pi^*(\tilde{e})]\}\Big\}(e - \underline{e})\ \mathrm{d}F(e) = 0.$$

In words, $g(x)$ has an inverted bell-shape bounded from above by zero at $x = \underline{e}$ and $x = \overline{e}$. Hence, $g(x) < 0$ for all $x \in (\underline{e}, \overline{e})$.

In particular, we have $g[p_1/f'(k_0^*)] < 0$ and thus $\mathrm{Cov}\{u'[\pi^*(\tilde{e})], \max[\tilde{e}f'(k_0^*) - p_1, 0]\} < 0$. Eq. (29) implies that $\mathrm{E}(\tilde{e})f'(k_0^*) < p_0 + \mathrm{E}\{\max[\tilde{e}f'(k_0^*) - p_1, 0]\}$. It then follows from Eq. (20) and the strict concavity of $f(k)$ that $k_0^* > k_0^c$. From Eq. (3), we have $k_0^* > k_0^c + k_1(e, k_0^c)$ for all $e < p_1/f'(k_0^*)$ and $k_0^* + k_1(e, k_0^*) = k_0^c + k_1(e, k_0^c)$ for all $e \geq p_1/f'(k_0^*)$. $\square$

References

Adam-Müller, A.F.A., 1997. Export and hedging decisions under revenue and exchange rate risk: A note. European Economic Review 41, 1421–1426.

Allayannis, G., Ihrig, J., Weston, J.P., 2001. Exchange-rate hedging: Financial versus operational strategies. American Economic Review Papers and Proceedings 91, 391–395.

Anand, J., Kogut, B., 1997. Technological capabilities of countries, firm rivalry and foreign direct investment. Journal of International Business Studies 28, 445–465.

Bodnar, G.M., Hayt, G.S., Marston, R.C., 1998. 1998 Wharton survey of financial risk management by US non-financial firms. Financial Management 27, 70–91.

Broll, U., 1992. The effect of forward markets on multinational firms. Bulletin of Economic Research 44, 233–240.

Broll, U., Eckwert, B., 2006. Transparency in the foreign exchange market and the volume of international trade. Review of International Economics 14, in press.

Broll, U., Wong, K.P., Zilcha, I., 1999. Multiple currencies and hedging. Economica 66, 421–432.

Broll, U., Zilcha, I., 1992. Exchange rate uncertainty, futures markets and the multinational firm. European Economic Review 36, 815–826.

Chang, E.C., Wong, K.P., 2003. Cross-hedging with currency options and futures. Journal of Financial and Quantitative Analysis 38, 555–574.

Dixit, A.K., Pindyck, R.S., 1994. Investment under Uncertainty. Princeton University Press, Princeton, NJ.

Drees, B., Eckwert, B., 2003. Welfare effects of transparency in foreign exchange markets: The role of hedging opportunities. Review of International Economics 11, 453–463.

Eckwert, B., Zilcha, I., 2001. The value of information in production economies. Journal of Economic Theory 100, 172–186.

Eckwert, B., Zilcha, I., 2003. Incomplete risk sharing arrangements and the value of information. Economic Theory 21, 43–58.

Froot, K.A., Scharfstein, D.S., Stein, J.C., 1993. Risk management: Coordinating corporate investment and financing policies. Journal of Finance 48, 1629–1658.

Gollier, C., 2001. The Economics of Risk and Time. MIT Press, Cambridge, MA.

Hines, J.R., Jr., 2001. Tax sparing and direct investment in developing countries. In: Hines, J.R., Jr. (Ed.), International Taxation and Multinational Activity. University of Chicago Press, Chicago, IL., pp. 39–66.

Kim, Y.S., Mathur, I., Nam, J., 2006. Is operational hedging a substitute for or a complement to financial hedging? Journal of Corporate Finance 12, in press.

Kogut, B., 1983. Foreign direct investment as a sequential process. In: Kindleberger, C.P., Audretsch, D. (Eds.), The Multinational Corporation in the 1980s. MIT Press, Cambridge, MA., pp. 38–56.

Lien, D., Wang, Y., 2006. Cross hedging with futures and options: The effects of disappointment aversion. Journal of Multinational Financial Management 16, 16–26.

Merton, R.C., 1973. Theory of rational option pricing. Bell Journal of Economics and Management Science 4, 141–183.

Pantzalis, C., Simkins, B.J., Laux, P.A., 2001. Operational hedges and the foreign exchange exposure of U.S. multinational corporations. Journal of International Business Studies 32, 793–812.

Pratt, J.W., 1964. Risk aversion in the small and in the large. Econometrica 32, 122–136.

Smith, C.W., Stulz, R.M., 1985. The determinants of firms' hedging policies. Journal of Financial and Quantitative Analysis 20, 391–405.

Stulz, R.M., 1984. Optimal hedging policies. Journal of Financial and Quantitative Analysis 19, 127–140.

Stulz, R.M., 1990. Managerial discretion and optimal financial policies. Journal of Financial Economics 26, 3–27.

Tufano, P., 1996. Who manages risk? An empirical examination of risk management practices in the gold mining industry. Journal of Finance 51, 1097–1137.

Wong, K.P., 2003a. Currency hedging with options and futures. European Economic Review 47, 833–839.

Wong, K.P., 2003b. Export flexibility and currency hedging. International Economic Review 44, 1295–1312.