International Tenders and Futures Hedging *

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International Tenders and Futures Hedging

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This paper examines the optimal bidding and hedging decisions of a risk-averse firm that takes part in an international tender. The firm faces multiple sources of uncertainty: exchange rate risk, risk of an unsuccessful tender, and business risk. The firm is allowed to trade unbiased currency futures contracts to imperfectly hedge its contingent foreign exchange risk exposure. We show that the firm shorts less (more) of the unbiased futures contracts when its marginal utility function is convex (concave) as compared with the case that the marginal utility function is linear. We further show that the curvature of the marginal utility function plays a decisive role in determining the impact of currency futures hedging on the firm's bidding behavior. Sufficient conditions that ensure the firm bids more or less aggressively than in the case without hedging opportunities are derived.

INTRODUCTION

A bidding firm that takes part in an international tender encounters multiple sources of uncertainty. For example, there is exchange rate risk and risk of an unsuccessful tender. Even in the case of a successful tender, the actual amount received by the firm may deviate significantly from the amount bid due to business risk. While exchange rate risk is readily hedgeable by trading currency derivatives, other sources of uncertainty are not. The purpose of this paper is therefore to study the optimal bidding and hedging decisions of the firm in general, and the effect of currency futures hedging on the firm's bidding behavior in particular.

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We develop an expected utility model wherein the firm is allowed to trade unbiased currency futures contracts for hedging purposes. The auction outcome is specified in reduced form by a subjective probability of success that is a decreasing function of the amount bid. The firm also faces business risk in that the actual amount received from the successful tender is subject to a multiplicative shock. Within this model, we show that the firm's optimal futures position depends crucially on the curvature of the firm's marginal utility function, $U'(W)$, where W is the firm's end-of-period domestic currency wealth. Specifically, the firm shorts less (more) of the unbiased futures contracts when $U'(W)$ is convex (concave) as compared with the case that $U'(W)$ is linear.

The curvature of $U'(W)$ also plays a decisive role in determining the effect of currency futures hedging on the firm's optimal bidding decision. If $U'(W)$ is linear or concave, we show that a sufficient condition for the firm to bid more aggressively in response to the availability of currency futures hedging is that the firm has mean-variance-skewness preferences (see Poitras and Heaney, 1999). If $U'(W)$ is convex, we show that a sufficient condition for the firm to bid less aggressively in response to the availability of currency futures hedging is that $U'(W)W$ is convex. Thus, the effect of currency futures hedging on the firm's bidding behavior can be diametrically opposed depending on the curvature of $U'(W)$.

The literature on the hedging of contingent foreign exchange risk exposure is abound. For example, Eaker and Grant (1985), Kerkvliet and Moffet (1991), and Broll, Wong, and Zilcha (1999) examine the optimal hedging decisions with currency futures. Wong (2003a, b), Chang and Wong (2003), and Lien and Wong (2004) enlarge the set of hedging instruments to include also currency options. Steil (1993) and Persson and Trovik (2000) study a somewhat different issue on the optimal security design. Except Lien and Wong (2004), the extant literature takes the distribution of non-market events as exogenously given and thus focuses solely on the optimal hedging decisions. This paper, like Lien and Wong (2004), allows the probability of winning an international tender to be endogenously determined. We further introduce business risk to the bidding firm by making the actual amount received from the successful tender differ from the amount bid, which is absent in

Lien and Wong (2004).

The main objective of Lien and Wong (2004) is to formalize the intuitive assertion that currency options, being flexible, are likely to perform better than currency futures in terms of hedging against contingent foreign exchange risk exposure (see, e.g., Feiger and Jacquillat, 1979; Giddy, 1985; Agmon and Eldor, 1985; and Chance, 2004). However, Lien and Tse (2001) empirically document that currency futures are better than currency options when hedging downside risk of the British pound, the Deutsche mark, and the Japanese yen (see also Chang and Shanker, 1986). Furthermore, Benet and Luft (1995) provide evidence that the hedging effectiveness of futures contracts over options contracts improves once transaction costs and margin requirements are taken into account. In light of these empirical studies, currency futures contracts remain an important hedging instrument for firms engaging in international tenders. The economic implications of hedging with currency futures should offer useful guidelines for these firms in the environment of volatile exchange rate movements.

The rest of this paper is organized as follows. The next section delineates a one-period model of a risk-averse firm that takes part in an international tender. The subsequent section derives the firm's optimal hedging decision when a currency futures market is available. This section also examines the effect of currency futures hedging on the firm's optimal bidding decision. The penultimate section offers an empirical study. The final section concludes.

THE MODEL

Consider a one-period model of a firm that puts in a tender for a foreign construction project. At the beginning of the period, the firm bids an amount, B, on the project, where B is denominated in the foreign currency. We model the auction outcome in reduced form by a function, $\Pi(B)$, that denotes the subjective probability of success. Since a higher bid should be more likely to fail, we assume that $\Pi'(B) < 0$. The final decision of the tender is announced at the end of the period.[1]

The end-of-period spot exchange rate, $\tilde{S}$, which is expressed in units of the domestic currency per unit of the foreign currency, is not known to the firm at the time when the bidding decision is made.[2] While the winning bidder bids B on the project, the actual amount received, $\tilde{\zeta}B$, is subject to a multiplicative shock, $\tilde{\zeta}$, that is non-negative and has a mean of unity.[3] We can regard this shock as driven by the randomness in the firm's profit margin from the project. Since there is no a priori relationship between the two random variables, $\tilde{S}$ and $\tilde{\zeta}$, we assume that they are independent for simplicity.

To hedge the exchange rate risk, the firm can trade infinitely divisible currency futures contracts with a contract size set equal to one unit of the foreign currency. The beginning-of-period futures price is predetermined at S_F, which is expressed in units of the domestic currency per unit of the foreign currency. Let Z be the number of the currency futures contracts purchased (sold if negative) by the firm at the beginning of the period. The firm's end-of-period wealth is thus given by

$$\tilde{W}_\alpha = \alpha\tilde{S}\tilde{\zeta}B + (\tilde{S} - S_F)Z, \tag{1}$$

where $\alpha = 1$ or 0 depending on whether the bid succeeds or fails, respectively.

The firm possesses a von Neumann-Morgenstern utility function, $U(W)$, defined over its end-of-period wealth, W, with $U'(W) > 0$ and $U''(W) < 0$, indicating the presence of risk aversion. The firm's ex-ante decision problem is to choose a bid, B, and a futures position, Z, so as to maximize the expected utility of its end-of-period wealth:

$$\max_{B,\ Z}\ \Pi(B)\mathrm{E}[U(\tilde{W}_1)] + [1 - \Pi(B)]\mathrm{E}[U(\tilde{W}_0)], \tag{2}$$

where $\mathrm{E}(\cdot)$ is the expectation operator, and $\tilde{W}_1$ and $\tilde{W}_0$ are defined in Equation (1) for $\alpha = 1$ and 0, respectively.

SOLUTION TO THE MODEL

The first-order conditions for program (2) are given by

$$\Pi(B^*)\mathrm{E}[U'(\tilde{W}_1^*)\tilde{S}\tilde{\zeta}] + \Pi'(B^*)\{\mathrm{E}[U(\tilde{W}_1^*)] - \mathrm{E}[U(\tilde{W}_0^*)]\} = 0, \tag{3}$$

and

$$\Pi(B^*)\mathrm{E}[U'(\tilde{W}_1^*)(\tilde{S} - S_F)] + [1 - \Pi(B^*)]\mathrm{E}[U'(\tilde{W}_0^*)(\tilde{S} - S_F)] = 0, \tag{4}$$

where Equation (3) is the first-order condition with respect to B, Equation (4) is the first-order condition with respect to Z, and an asterisk (*) indicates an optimal level.[4]

Optimal Hedging Decisions

The firm's optimal futures position, Z^*, is implicitly defined in Equation (4). Among other things, Z^* would depend on whether the firm perceives the beginning-of-period futures price, S_F, as unbiased or biased. In the former unbiased case, Z^* reflects solely the hedging motive of the firm. In the latter biased case, Z^* reflects also the speculative motive of the firm. To focus on the impact of currency futures hedging on the firm's bidding behavior due entirely to the risk-sharing effect but not to the wealth effect, we assume hereafter that the beginning-of-period futures price is perceived as unbiased by the firm. That is, we assume that $S_F = \mathrm{E}(\tilde{S})$.

Using the covariance operator, $\mathrm{Cov}(\cdot,\ \cdot)$, we can write Equation (4) as[5]

$$\mathrm{E}[\Phi(\tilde{S})][\mathrm{E}(\tilde{S}) - S_F] + \mathrm{Cov}[\Phi(\tilde{S}),\ \tilde{S}] = 0, \tag{5}$$

where $\Phi(S) = \Pi(B^*)\mathrm{E}[U'(\tilde{W}_1^*)|S] + [1 - \Pi(B^*)]\mathrm{E}[U'(\tilde{W}_0^*)|S]$ and $\mathrm{E}(\cdot|\cdot)$ is the conditional expectation operator. By the unbiasedness assumption, the first-term on the left-hand side of Equation (5) vanishes and thus we have

$$\mathrm{Cov}[\Phi(\tilde{S}),\ \tilde{S}] = 0. \tag{6}$$

The following proposition shows that the optimal futures position depends crucially on whether the firm's marginal utility function is convex or concave (all proofs of propositions are relegated to Appendix A).

Proposition 1:

Suppose that the beginning-of-period futures price is perceived as unbiased by the firm. If the firm's utility function exhibits $U'''(W) \geq (\leq)\ 0$, then the optimal futures position, Z^*, satisfies $Z^* \geq (\leq) - \Pi(B^*)B^*$, where the equality holds only when $U'''(W) \equiv 0$.

To see the intuition of Proposition 1, note that covariances can be interpreted as marginal variances. Inspection of Equation (6) reveals that the firm's optimal futures position, Z^*, minimizes the variance of the expected marginal utility across different realizations of $\tilde{S}$. Substituting $Z = -\Pi(B)B$ into Equation (1) yields the firm's end-of-period wealth:

$$\tilde{W}_\alpha = S_F \Pi(B)B + \tilde{S}[\alpha\tilde{\zeta} - \Pi(B)]B. \tag{7}$$

If the firm's utility function is quadratic (i.e., $U'''(W) \equiv 0$), the marginal utility function becomes linear. The variability of the expected marginal utility across different realizations of $\tilde{S}$ as such comes entirely from the variability of the firm's expected end-of-period wealth. It follows from Equation (7) that

$$\mathrm{Cov}\{\Pi(B)\mathrm{E}(\tilde{W}_1|\tilde{S}) + [1 - \Pi(B)]\mathrm{E}(\tilde{W}_0|\tilde{S}),\ \tilde{S}\} = \mathrm{Cov}[S_F\Pi(B)B,\ \tilde{S}] = 0. \tag{8}$$

Thus, Equations (6) and (8) imply that $Z^* = -\Pi(B^*)B^*$ is indeed optimal for the firm with a quadratic utility function.[6]

If the firm's utility function exhibits $U'''(W) \geq (\leq)\ 0$, then according to Kimball (1990, 1993) the firm is more sensitive to low (high) realizations of its end-of-period wealth than to high (low) ones. The firm as such has incentives to avoid the low (high) realizations of its end-of-period wealth, which occur when the bid fails (succeeds) and the end-of-period spot exchange rate is high. To this end, a long (short) futures position is called for. Since the firm with a quadratic utility function would optimally sell $\Pi(B^*)B^*$ currency futures contracts, introducing convexity (concavity) to the firm's marginal utility function induces the firm to decrease (increase) its short futures position, thereby rendering $Z^* \geq (\leq) -\Pi(B^*)B^*$ optimal.

Optimal Bidding Decisions

Consider the benchmark case wherein the firm is not allowed to hedge its exchange rate risk exposure. In this case, the firm encounters the following ex-ante decision problem:

$$\max_B \; EU = \Pi(B)\mathrm{E}[U(\tilde{S}\tilde{\zeta}B)] + [1 - \Pi(B)]U(0). \tag{9}$$

The first-order condition for program (9) is given by

$$\Pi(B^{**})\mathrm{E}[U'(\tilde{S}\tilde{\zeta}B^{**})\tilde{S}\tilde{\zeta}] + \Pi'(B^{**})\{\mathrm{E}[U(\tilde{S}\tilde{\zeta}B^{**})] - U(0)\} = 0, \tag{10}$$

where B^{**} is the optimal bid in the absence of currency futures hedging.

Differentiating EU as defined in program (9) with respect to B and evaluating at $B = B^*$ yields

$$\left.\frac{\mathrm{d}EU}{\mathrm{d}B}\right|_{B=B^*} = \Pi(B^*)\mathrm{E}[U'(\tilde{S}\tilde{\zeta}B^*)\tilde{S}\tilde{\zeta}] + \Pi'(B^*)\{\mathrm{E}[U(\tilde{S}\tilde{\zeta}B^*)] - U(0)\}. \tag{11}$$

Subtracting Equation (3) from the right-hand side of Equation (11) yields

$$\begin{aligned}\left.\frac{\mathrm{d}EU}{\mathrm{d}B}\right|_{B=B^*} &= \Pi(B^*)\{\mathrm{E}[U'(\tilde{S}\tilde{\zeta}B^*)\tilde{S}\tilde{\zeta}] - \mathrm{E}[U'(\tilde{W}_1^*)\tilde{S}\tilde{\zeta}]\} \\ &\quad + \Pi'(B^*)\{\mathrm{E}[U(\tilde{S}\tilde{\zeta}B^*)] - U(0) - \mathrm{E}[U(\tilde{W}_1^*)] + \mathrm{E}[U(\tilde{W}_0^*)]\}. \end{aligned} \tag{12}$$

If the right-hand side of Equation (12) is positive (negative), Equation (10) and the second-order condition for program (9) imply that $B^* < (>)\ B^{**}$.

In general, the sign of the right-hand side of Equation (12) is indeterminate without imposing strong conditions on $U(W)$. To derive some concrete results, we follow Poitras and

Heaney (1999) to consider the case wherein the firm has mean-variance-skewness preferences of the generic form:

$$U(W) = U(0) + aW - \frac{b}{2}W^2 + \frac{c}{6}W^3, \tag{13}$$

where a, b, and c are constants with $a > 0$ and $b > 0$. The following proposition characterizes sufficient conditions under which the right-hand side of Equation (12) is unambiguously negative.

Proposition 2:

Suppose that the beginning-of-period futures price is perceived as unbiased by the firm. If the firm has mean-variance-skewness preferences represented by Equation (13) with $c \leq 0$, then it is optimal for the firm to raise its bid in the presence of currency futures hedging than in the absence of currency futures hedging, i.e., $B^* > B^{**}$.

The result of Proposition 2 matches the conventional wisdom that the availability of risk-sharing opportunities should make the firm behave as if it is less risk averse. The firm as such would bid more aggressively in the presence of currency futures hedging in that its optimal bid moves closer to the optimal bid under risk neutrality.

Given that $U(W)$ is defined in Equation (13), we have $U'''(W) = c$. Thus, Proposition 2 says that a sufficient condition for $B^* > B^{**}$ is that $U'''(W) \leq 0$. However, Kimball (1990, 1993) convincingly argues that prudence, i.e., $U'''(W) \geq 0$, is a more reasonable behavioral assumption for decision making under multiple sources of uncertainty. Intuitively, prudence measures the propensity to prepare and forearm oneself under uncertainty, vis-à-vis risk aversion that is how much one dislikes uncertainty and would turn away from it if one could. As shown by Leland (1968), Drèze and Modigliani (1972), and Kimball (1990), prudence is both necessary and sufficient to generate precautionary saving. Furthermore, prudence is implied by decreasing absolute risk aversion, which is instrumental in yielding many intuitively appealing comparative statics under uncertainty (Gollier, 2001). Since the sufficient conditions in Proposition 2 rule out the reasonable case of prudence (i.e., $c > 0$), the usefulness of these conditions are skeptical.

To derive sufficient conditions that can determine the sign of the right-hand side of Equation (12) and at the same time allow for prudence, we assume away the multiplicative shock by setting $\tilde{\zeta} \equiv 1$ for tractability. Let $H(S)$ be the cumulative distribution of $\tilde{S}$ over support $[\underline{S},\ \overline{S}]$, where $0 < \underline{S} < \overline{S} < \infty$. The following proposition shows that the convexity of $U'(W)W$, which implies prudence, i.e., $U'''(W) \geq 0$, is among the sufficient conditions that ensure the right-hand side of Equation (12) to be positive.

Proposition 3:

Suppose that the beginning-of-period futures price is perceived as unbiased by the firm and that the cash flow from the tender is certain, i.e., $\tilde{\zeta} \equiv 1$. If the firm's utility function, $U(W)$, satisfies that $U'(W)W$ is convex for all $W \in [\underline{S}B^*,\ \overline{S}B^*]$, then it is optimal for the firm to lower its bid in the presence of currency futures hedging than in the absence of currency futures hedging, i.e., $B^* < B^{**}$.

To show that the sufficient conditions characterized in Proposition 3 are relatively general, we consider the following utility function proposed by Bell and Fishburn (2000): $U(W) = \alpha W - \beta e^{-\gamma W}$, where α, β, and γ are positive scalars. Given this utility function, $U'(W)W$ is convex for all $W \in [\underline{S}B^*,\ \overline{S}B^*]$ if $U''(W)(2-\gamma W) \geq 0$, for all $W \in [\underline{S}B^*,\ \overline{S}B^*]$. When α is sufficient large relative to β, the firm behaves as if it were risk neutral. Thus, B^* is large (and close to the risk-neutral optimal bid), so is $\underline{S}B^*$. Hence, for γ sufficiently large such that $\gamma\underline{S}B^* \geq 2$, the firm with this utility function optimally bids more aggressively in the absence of currency futures hedging than in the presence of currency futures hedging.

AN EMPIRICAL STUDY

Consider an Australian firm that bids for a construction project in the United States. The firm has a Bell-Fishburn utility function: $U(W) = W - \beta e^{-\gamma W}$, where β and γ are positive scalars. The probability of winning the tender is described by $\Pi(B) = 1/(1+B)^\theta$, where $\theta > 1$. Daily data on spot exchange rates for the Australian dollar against the U.S. dollar over the period from January 1, 2000 to December 31, 2001 (a total of 521 observations) are supplied by Datastream. Table 1 tabulates the firm's optimal bidding and hedging decisions for several combinations of parameter values.

Table 1. Optimal bidding and hedging decisions[a]

β	γ	θ	B^*	Z^*	U^*	B^{**}	U^{**}
5	2	1.5	0.4726	−0.0897	−1156	0.8062	−1879
5	5	1.5	0.2625	−0.0489	−761	0.5009	−1469
10	2	1.5	0.4385	−0.0978	−2558	0.7779	−3853
5	2	2.5	0.3148	−0.0781	−1556	0.4033	−2152

[a]The firm has a Bell-Fishburn utility function: $U(W) = W - \beta e^{-\gamma W}$, where β and γ are positive scalars. The probability of winning the tender is given by $\Pi(B) = 1/(1+B)^\theta$, where $\theta > 1$. U^* is the maximum expected utility level corresponding to B^* and Z^*. U^{**} is the maximum expected utility level corresponding to B^{**}.

It is evident from Table 1 that $B^* < B^{**}$ in all cases. The improvement in the expected utility level due to the presence of currency futures hedging is significant, ranging from 28% to 48%. A few findings are in order. First, B^* and B^{**} decrease and Z^* increases when θ increases. A larger θ implies that the probability of winning the tender declines faster with an increase in the bid. The firm becomes more conservative and thereby lowers its bid and reduces its short futures position. Second, B^* and B^{**} decrease and Z^* increases when γ increases. That is, a more risk-averse firm bids more conservatively and shorts fewer futures contracts. Finally, B^*, B^{**}, and Z^* all decrease when β increases. Herein the firm is more concerned about the risk-averse component of its preferences and thus reduces its bids. The increase in β, however, promotes the marginal benefits of hedging, thereby leading to an increase in the firm's short futures position.

CONCLUSIONS

This paper has examined the optimal bidding and hedging decisions of a risk-averse firm that takes part in an international tender. The firm faces three sources of uncertainty: exchange rate risk, risk of an unsuccessful tender, and business risk. The firm is allowed to trade unbiased currency futures contracts to imperfectly hedge its contingent foreign exchange risk exposure. Within this expected utility model, we have shown that the curvature of the firm's marginal utility function, $U'(W)$, where W is the firm's end-of-period domestic currency wealth, plays a decisive role in determining the firm's optimal bidding and hedging decisions. Specifically, the firm shorts less (more) of the unbiased futures contracts when $U'(W)$ is convex (concave) as compared with the case that $U'(W)$ is linear. If $U'(W)$ is linear or concave and the firm has mean-variance-skewness preferences (see Poitras and Heaney, 1999), the availability of currency futures hedging induces the firm to bid more aggressively. On the other hand, if $U'(W)W$ is convex and thus $U'(W)$ is convex, the availability of currency futures hedging induces the firm to bid less aggressively. Thus, the effect of currency futures hedging on the firm's bidding behavior can be diametrically opposed depending on the curvature of $U'(W)$.

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APPENDIX A

Proof of Proposition 1

Differentiating $\Phi(S)$ with respect to S yields

$$\Phi'(S) = \Pi(B^*)\mathrm{E}[U''(\tilde{W}_1^*)(\tilde{\zeta}B^* + Z^*)|S] + [1 - \Pi(B^*)]\mathrm{E}[U''(\tilde{W}_0^*)Z^*|S]$$

$$= \mathrm{E}[U''(\tilde{W}_1^*)|S][\Pi(B^*)B^* + Z^*] + \mathrm{E}[U''(\tilde{W}_0^*) - U''(\tilde{W}_1^*)|S][1 - \Pi(B^*)]Z^*$$

$$+\Pi(B^*)B^*\mathrm{Cov}[U''(\tilde{W}_1^*),\ \tilde{\zeta}|S]. \tag{A.1}$$

If $U'''(W) \geq 0$, it follows from $W_1^* - W_0^* = S\zeta B^* \geq 0$ that $U''(W_1^*) \geq U''(W_0^*)$. Also, we have $\mathrm{Cov}[U''(\tilde{W}_1^*),\ \tilde{\zeta}|S] \geq 0$ as $\partial U''(W_1^*)/\partial\zeta = U'''(W_1^*)SB^* \geq 0$. Suppose that $Z^* \leq -\Pi(B^*)B^*$. Then, Equation (A.1) implies that $\Phi'(S)$ is non-negative and thus the left-hand side of Equation (6) is positive. Hence, for Equation (6) to hold, it must be true that $Z^* > -\Pi(B^*)B^*$.

Now, if $U'''(W) \leq 0$, we have $U''(W_1^*) \leq U''(W_0^*)$ and $\mathrm{Cov}[U''(\tilde{W}_1^*),\ \tilde{\zeta}|S] \leq 0$. Suppose that $-\Pi(B^*)B^* \leq Z^*$. Then, Equation (A.1) implies that $\Phi'(S)$ is non-positive and thus the left-hand side of Equation (6) is negative. Hence, for Equation (6) to hold, it must be true that $Z^* < -\Pi(B^*)B^*$.

Finally, if $U'''(W) \equiv 0$, then $U''(W_1^*) = U''(W_0^*)$ and $\mathrm{Cov}[U''(\tilde{W}_1^*),\ \tilde{\zeta}|S] = 0$. When $Z^* = -\Pi(B^*)B^*$, Equation (A.1) implies that $\Phi'(S) = 0$ and thus Equation (6) holds.

Proof of Proposition 2

Rearranging terms of Equation (12) yields

$$\left.\frac{\mathrm{d}EU}{\mathrm{d}B}\right|_{B=B^*} = \left[\frac{\Pi(B^*)}{B^*} + \Pi'(B^*)\right]\{\mathrm{E}[U(\tilde{S}\tilde{\zeta}B^*)] - U(0) - \mathrm{E}[U(\tilde{W}_1^*)] + \mathrm{E}[U(\tilde{W}_0^*)]\}$$

$$+\frac{\Pi(B^*)}{B^*}\{\mathrm{E}[U'(\tilde{S}\tilde{\zeta}B^*)\tilde{S}\tilde{\zeta}B^*] - \mathrm{E}[U'(\tilde{W}_1^*)\tilde{S}\tilde{\zeta}B^*]$$

$$+\mathrm{E}[U(\tilde{W}_1^*)] - \mathrm{E}[U(\tilde{W}_0^*)] - \mathrm{E}[U(\tilde{S}\tilde{\zeta}B^*)] + U(0)\}. \tag{A.2}$$

Since $(B^*,\ Z^*)$ is the optimal solution to program (2), it must be true that

$$\Pi(B^*)\mathrm{E}[U(\tilde{W}_1^*)] + [1-\Pi(B^*)]\mathrm{E}[U(\tilde{W}_0^*)] > \Pi(B^*)\mathrm{E}[U(\tilde{S}\tilde{\zeta}B^*)] + [1-\Pi(B^*)]U(0).$$

Rearranging terms yields

$$\mathrm{E}[U(\tilde{W}_1^*)] - \mathrm{E}[U(\tilde{W}_0^*)] - \mathrm{E}[U(\tilde{S}\tilde{\zeta}B^*)] + U(0) > \frac{U(0) - \mathrm{E}[U(\tilde{W}_0^*)]}{\Pi(B^*)} > 0, \tag{A.3}$$

where the second inequality follows from $\mathrm{E}(\tilde{W}_0^*) = [\mathrm{E}(\tilde{S}) - S_F]Z^* = 0$, $U''(W) < 0$, and Jensen's inequality. It is well known that the firm's optimal bid, B^c, is higher when the firm is risk neutral, where B^c solves $\Pi(B^c) + \Pi'(B^c)B^c = 0$. Thus, we have $\Pi(B^*)/B^* > \Pi'(B^*)$. It then follows from inequality (A.3) that the first term on the right-hand side of Equation (A.2) is unambiguously negative. The sign of the second-term on the right-hand side of Equation (A.2) is, however, indeterminate without imposing additional restrictions on $U(W)$.

Given that the firm has mean-variance-skewness preferences of the generic form: $U(W) = U(0) + aW - bW^2/2 + cW^3/6$. Applying the Taylor series expansion around $S\zeta B^*$ yields

$$U'(W_1^*) = U'(S\zeta B^*) + U''(S\zeta B^*)(S - S_F)Z^* + c(S - S_F)^2\frac{Z^{*2}}{2}, \tag{A.4}$$

since $U'''(W) = c$. Multiplying $S\zeta B^*$ to both side of Equation (A.4) and taking expectations yields

$$\mathrm{E}[U'(\tilde{W}_1^*)\tilde{S}\tilde{\zeta}B^*] = \mathrm{E}[U'(\tilde{S}B^*)\tilde{S}\tilde{\zeta}B^*] - \left(b + \frac{c}{2}S_F Z^*\right)B^*Z^*\mathrm{Var}(\tilde{S})$$

$$+\frac{c}{2}B^*Z^*\{2[1+\mathrm{Var}(\tilde{\zeta})]B^* + Z^*\}\mathrm{Cov}(\tilde{S},\ \tilde{S}^2), \tag{A.5}$$

since $U''(W) = -b + cW$. Applying the Taylor series expansion around $S\zeta B^*$ yields

$$U(W_1^*) = U(S\zeta B^*) + U'(S\zeta B^*)(S - S_F)Z^* + U''(S\zeta B^*)(S - S_F)^2 \frac{Z^{*2}}{2}$$

$$+c(S - S_F)^3 \frac{Z^{*3}}{6}, \tag{A.6}$$

Taking expectations on both sides of Equation (A.6) yields

$$\mathrm{E}[U(\tilde{W}_1^*)] = \mathrm{E}[U(\tilde{S}\tilde{\zeta}B^*)] - \left(b + \frac{c}{2}S_F Z^* + \frac{bZ^*}{2B^*}\right) B^* Z^* \mathrm{Var}(\tilde{S})$$

$$+\frac{c}{2}B^* Z^* \{[1 + \mathrm{Var}(\tilde{\zeta})]B^* + Z^*\}\mathrm{Cov}(\tilde{S},\ \tilde{S}^2) + \frac{c}{6}Z^{*3}\mathrm{E}[(\tilde{S} - S_F)^3], \tag{A.7}$$

since $U'(W) = a - bW + cW^2/2$ and $U''(W) = -b + cW$. Applying the Taylor series expansion around zero yields

$$U(W_0^*) = U(0) + U'(0)(S - S_F)Z^* + U''(0)(S - S_F)^2 \frac{Z^{*2}}{2} + c(S - S_F)^3 \frac{Z^{*3}}{6}, \tag{A.8}$$

Taking expectations on both sides of Equation (A.8) yields

$$\mathrm{E}[U(\tilde{W}_0^*)] = U(0) - \frac{b}{2}Z^{*2}\mathrm{Var}(\tilde{S}) + \frac{c}{6}Z^{*3}\mathrm{E}[(\tilde{S} - S_F)^3], \tag{A.9}$$

since $U'(0) = a$ and $U''(0) = -b$. Substituting Equations (A.5), (A.7), and (A.9) into the second term on the right-hand side of Equation (A.2) yields

$$-\Pi(B^*)B^* Z^* \frac{c}{2}[1 + \mathrm{Var}(\tilde{\zeta})]\mathrm{Cov}(\tilde{S},\ \tilde{S}^2). \tag{A.10}$$

Since $\mathrm{Cov}(\tilde{S},\ \tilde{S}^2) > 0$ and $Z^* < 0$, it follows from expression (A.10) that the second term on the right-hand side of Equation (A.2) is unambiguously negative whenever $c \leq 0$. Thus, in this case, we have $B^* > B^{**}$.

Proof of Proposition 3

Let $\tilde{X} = \tilde{S}B^*$ and $F(X)$ be the cumulative distribution function of $\tilde{X}$. Using the change-of-variable technique (Hogg and Craig, 1989), $\tilde{X}$ has support $[\underline{S}B^*,\ \overline{S}B^*]$ and $F(X) = H(X/B^*)$. Thus,

$$\mathrm{E}[U'(\tilde{S}B^*)\tilde{S}B^*] = \int_{\underline{S}B^*}^{\overline{S}B^*} U'(X)X\ \mathrm{d}F(X). \tag{A.11}$$

Likewise, let $G(W_1^*)$ be the cumulative distribution function of $\tilde{W}_1^* = \tilde{S}B^* + (\tilde{S} - S_F)Z^*$. Using the change-of-variable technique, $\tilde{W}_1^*$ has support $[\underline{S}B^* + (\underline{S} - S_F)Z^*,\ \overline{S}B^* + (\overline{S} - S_F)Z^*]$ and $G(W_1^*) = F[(W_1^* + S_F Z^*)B^*/(B^* + Z^*)]$. Thus,

$$\mathrm{E}[U'(\tilde{W}_1^*)\tilde{W}_1] = \int_{\underline{S}B^* + (\underline{S}-S_F)Z^*}^{\overline{S}B^* + (\overline{S}-S_F)Z^*} U'(X)X\ \mathrm{d}G(X). \tag{A.12}$$

Subtracting Equation (A.11) by Equation (A.12) yields

$$\mathrm{E}[U'(\tilde{S}B^*)\tilde{S}B^*] - \mathrm{E}[U'(\tilde{W}_1^*)\tilde{W}_1] = \int_{\underline{S}B^*}^{\overline{S}B^*} U'(X)X\ \mathrm{d}[F(X) - G(X)], \tag{A.13}$$

which follows from the fact that $\mathrm{d}G(X) = 0$ for all $X \in [\underline{S}B^*,\ \underline{S}B^* + (\underline{S} - S_F)Z^*] \bigcup [\overline{S}B^* + (\overline{S} - S_F)Z^*,\ \overline{S}B^*]$.

Since $G(X) = F[(X + S_F Z^*)B^*/(B^* + Z^*)]$, we have $F(X) - G(X) > (<)\ 0$ whenever $X < (>) S_F B^*$. That is, there is a single-crossing of $F(X)$ from below by $G(X)$ at $X = S_F B^*$. Also, we have

$$\int_{\underline{S}B^*}^{\overline{S}B^*} F(X)\ \mathrm{d}X = \int_{\underline{S}B^*}^{\overline{S}B^*} G(X)\ \mathrm{d}X = (\overline{S} - S_F)B^*.$$

Thus, $F(X)$ is a mean preserving spread of $G(X)$ in the sense of Rothschild and Stiglitz (1970). According to Rothschild and Stiglitz (1971), if $U'(W)W$ is convex for all $W \in [\underline{S}B^*,\ \overline{S}B^*]$, the right-hand side of Equation (A.13) is unambiguously positive:

$$\mathrm{E}[U'(\tilde{S}B^*)\tilde{S}B^*] - \mathrm{E}[U'(\tilde{W}_1^*)\tilde{W}_1^*] \geq 0. \tag{A.14}$$

Rewriting inequality (A.14) yields

$$\{\mathrm{E}[U'(\tilde{S}B^*)\tilde{S}] - \mathrm{E}[U'(\tilde{W}_1^*)\tilde{S}]\}B^* - \mathrm{Cov}[U'(\tilde{W}_1^*),\ \tilde{S}]Z^* \geq 0, \tag{A.15}$$

if $U'(W)W$ is convex for all $W \in [\underline{S}B^*,\ \overline{S}B^*]$. Since the convexity of $U'(W)W$ implies that $U'''(W) \geq 0$, we have $Z^* \geq -\Pi(B^*)B^*$ from Proposition 1. Since $\partial U'(W_1^*)/\partial S = U''(W_1^*)(B^* + Z^*) < 0$, we have $\mathrm{Cov}[U'(\tilde{W}_1^*),\ \tilde{S}] < 0$. Inequality (A.15) then implies that $\mathrm{E}[U'(\tilde{S}B^*)\tilde{S}] > \mathrm{E}[U'(\tilde{W}_1^*)\tilde{S}]$ and thus the second term on the right-hand side of Equation (12) is unambiguously positive. From inequality (A.3) and $\Pi'(B) < 0$, the first term on the right-hand side of Equation (12) is also unambiguously positive. Thus, in this case, we have $B^* < B^{**}$.

NOTES

1. We restrict our attention to the partial equilibrium solution for one bidder. This is innocuous only when the universe of auction participants is composed of homogeneous bidders facing identical information sets.

2. Throughout the paper, random variables have a tilde ($\sim$) while their realizations do not.

3. None of the qualitative results are affected if the shock is additive in nature.

4. A sufficient (but not necessary) condition to ensure the second-order conditions is that $\Pi''(B) \leq 0$.

5. For any two random variables, $\tilde{X}$ and $\tilde{Y}$, we have $\mathrm{E}(\tilde{X}\tilde{Y}) = \mathrm{E}(\tilde{X})\mathrm{E}(\tilde{Y}) + \mathrm{Cov}(\tilde{X},\ \tilde{Y})$.

6. For the case of unbiased currency futures contracts, the optimal futures position when the firm has a quadratic utility function is also the minimum-variance futures position.

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