

Using an error-correction model to test whether endogenous long-run growth exists

by

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Abstract:

A major empirical interest in growth studies is whether permanent changes in economic fundamentals affect the long-run growth rate or not. However, a direct time series analysis of this hypothesis may not always be feasible because the permanence of many such changes is rather questionable. This paper explains why examining the long-run effects of temporary changes in investment share on per capita output provides indirectly the answer regarding the effects of (possibly hypothetical) permanent changes in investment share, when per capita output and per capita investment are cointegrated. Applying the proposed method to the post-war data of major industrial countries, it is found that a disturbance to investment share does not produce a positive long-run effect in each of the four countries—France, Italy, Japan and the UK—in which per capita output and per capita investment are cointegrated. The evidence is unfavorable to the class of endogenous growth models.

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1 Introduction

Whether a permanent change in economic fundamentals produces a change in the long-run growth rate of the economy is an empirical question that many economists and policy makers are interested in. Moreover, it is a distinguishing characteristic between endogenous and exogenous growth models because the change leads to a growth effect (i.e., a change in the long-run growth rate) in the former class of models but only a level effect in the latter; see, for example, Romer (1986) and Lucas (1988). Based on this implication, Jones (1995) performs empirical analysis and concludes that the evidence on major industrial countries is unfavorable to the class of endogenous growth models. Similarly, Stokey and Rebelo (1995) conclude that income taxes do not have a growth effect according to the evidence provided by the tax reform ‘experiment’ in the United States of America (USA): income tax revenues increased dramatically from 2% to 15% of output in the early 1940s, but there was no change in per capita output growth.

While the endogenous and exogenous growth models imply different long-run effects of permanent changes in economic fundamentals, a direct examination of this hypothesis may not always be feasible because *the permanence of many such changes is rather questionable*. As an example, one of the frequently cited evidence against endogenous growth models is that the growth rates of per capita output (in USA and other industrial countries after World War II) are essentially trendless, but many investment share (i.e., investment-output ratio) series, based on total investment or producer durables investment, contain either strong positive trends or unit roots (Jones, 1995, Table IV). While the evidence regarding stationary output growth is expected, the conclusion of non-stationary investment shares in many industrial countries is quite different from those in several well-known empirical studies such as King et al. (1991). Moreover, the stationarity of some ‘great ratios’ such as the consumption-output ratio and investment-output ratio is regarded by many researchers as a stylized fact; see King et al. (1991) and especially Cochrane (1994). One may expect that many economists and econometricians, trained to be critical, would demand more evidence before deciding whether the endogenous growth models are empirically relevant or not.

By assuming explicitly that a permanent change in investment share is absent (or at least cannot be established affirmatively) in the data, this paper takes a complementary approach to deal with the question regarding the presence or absence of a growth effect of a (possibly hypothetical) permanent change. It examines the long-run effect of a temporary change in investment share on per capita output, and explains why the proposed method provides indirectly the answer to the above question, when *per capita output and per capita investment are cointegrated*.

The connection between these two apparently distinct questions (long-run effects of permanent and temporary changes in investment share respectively) is implied by the theoretical results on the time series properties of stochastic endogenous and exogenous growth models. Lau (1997) shows that permanent changes in economic fundamentals lead to growth effects, and temporary changes cause permanent level effects for endogenous growth models. On the other hand, permanent changes in economic fundamentals lead only to level effects, and temporary changes may cause either permanent or temporary level effects for exogenous growth models.¹ Moreover, Lau (1999, p. 18) points out a systematic difference between endogenous and exogenous growth models in terms of the long-run effect of a temporary change in economic fundamentals when the observed variables are cointegrated. Specifically, for a system of n variables with r ($1 \leq r \leq n - 1$) cointegrating vectors, the long-run multiplier matrix for the structural vector moving-average (VMA) representation, which summarizes the long-run effects of the structural disturbances on the level of observed variables, is of reduced rank of $n - r$. If this cointegrated system is generated by an exogenous growth model, then exactly r columns of the structural long-run multiplier matrix are zero. On the other hand, if this cointegrated system is generated by an endogenous growth model, then there is no column of zero in the structural long-run multiplier matrix.²

This paper exploits the above systematic difference between endogenous and exogenous growth models and examines the long-run effect of a disturbance to investment share on per capita output. An important paper, Levine and Renelt (1992), examines the results of cross-section growth studies (such as Barro, 1991; Mankiw et al., 1992) and finds that most ‘statistically significant’ regression results are fragile with respect to minor changes in specification. However, they do identify a positive and robust correlation between economic

¹ Kocherlakota and Yi (1996, p. 132) mention that “In exogenous growth models temporary policy changes do not permanently affect GNP, while in endogenous growth models temporary policy changes can permanently affect GNP.” As made clear in (21) of Lau (1997), a temporary change in economic fundamentals in exogenous growth models may produce either temporary effect (if the associated external impulse process is $I(0)$) or permanent effect (if the associated external impulse process is $I(1)$) on the level of output.

² In essence, a ‘reduced rank of $n - r$ (i.e., $n - r$ independent columns) in the structural long-run multiplier matrix’ and ‘ r columns of zero in this matrix’ are not necessarily the same. The first condition is related to the *number of independent cointegrating relationships*, while the second condition is related to the *zero long-run effect* of some structural disturbances. The first condition only implies that at most r structural disturbances produce zero long-run effect; it does not pin down the precise number of structural disturbances that produce zero long-run effect. However, this paper shows that testable implications appear when the cointegration feature is combined with the time series properties of stochastic endogenous and exogenous growth models.

growth and the share of investment in output. (This point is also mentioned in Mankiw, 1995, p. 302.) It is interesting to investigate whether this relationship is found using time series analysis.³

Since the long-run effects of temporary changes in investment share on per capita output for endogenous and exogenous growth models have testable implications when per capita output and per capita investment are cointegrated, the empirical method proposed in this paper is designed for a bivariate cointegrated system. The procedure is then applied to post-war data of several major industrial countries. It is found that a temporary change in investment share does not produce a positive long-run effect on per capita output in France, Italy, Japan and the United Kingdom (UK). The conclusions based on the time series studies of this paper, as well as those of Jones (1995), are quite different from the positive and significant relationship between investment share and output growth found in cross-section studies.

This paper is organized as follows. Section 2 summarizes the different properties of endogenous and exogenous growth models regarding the long-run effects of permanent and temporary changes in economic fundamentals respectively. Starting with the systematic difference between endogenous and exogenous growth models about the long-run multiplier matrix of the structural VMA representation (which summarizes the long-run effects of the underlying disturbances) when per capita output and per capita investment are cointegrated, Section 3 suggests a method to investigate whether a disturbance to investment share produces zero or positive long-run effect. Section 4 presents the empirical results. Section 5 relates the analysis of this paper to the growth literature and provides the conclusions.

2 Long-run effects of permanent and temporary changes in economic fundamentals

The long-run effects of permanent and temporary changes in economic fundamentals for stochastic endogenous and exogenous growth models have been examined in Lau (1997, 1999). To illustrate the relevant idea of these two papers in a framework which is useful to the subsequent empirical analysis, this

³ While different factors such as human capital, trade, government investment and tax rates have been emphasized in the growth literature (such as Lucas, 1988; Glomm and Ravikumar, 1994; Collard, 1999), the one variable which is present in most, if not all, growth models is investment in physical capital. Jones (1995, p. 505) suggests that a test based on investment data may, arguably, be regarded as providing evidence with respect to the endogenous versus exogenous growth debate for the whole class of growth models, rather than for a specific model only.

section studies, respectively, the stochastic Solow-Swan model (Solow, 1956; Swan, 1956) and its endogenous growth counterpart.

Consider first a stochastic version of the Solow-Swan model with exogenous technological progress. A closed economy is populated by a constant number (equal to labor input, N) of identical agents. The supply side of the economy is characterized by the following Cobb-Douglas production function of a representative agent,

$$Y_t = AK_t^\alpha \left[(1 + \tau)^t N \right]^{1-\alpha} e_t^P, \quad (1)$$

where $0 < \alpha < 1$, $\tau > 0$, Y_t is output at time t , K_t is capital input at t , and e_t^P is an impulse process to productivity. Parameter τ is the average rate of exogenous labor-augmenting technological progress.

The demand side of the economy is represented by:

$$\frac{I_t}{Y_t} = s e_t^I, \quad (2)$$

where I_t is investment at time t , s ($0 < s < 1$) is the average investment share in output, and e_t^I is an external impulse to the investment share at time t . Capital stock is assumed to depreciate at a constant rate of δ ($0 < \delta < 1$) per period.

The two mean-zero external impulse processes are assumed to have the following form:

$$(1 - L)^{c_j} q_j(L) \ln e_t^j = \varepsilon_t^j, \quad (3)$$

where L is the lag operator ($L \ln e_t^j = \ln e_{t-1}^j$), $j = P$ or I , parameter c_j is either 0 or 1, $q_j(L)$ contains only roots strictly outside the unit circle, and ε_t^P and ε_t^I are uncorrelated white noise structural disturbances.⁴ The impulse process $\ln e_t^j$ is I(0) when $c_j = 0$ or is I(1) without drift when $c_j = 1$.

Denote in lower case letter the corresponding variable per capita. In order for log per capita output and log per capita investment to be difference-stationary and cointegrated in this model, it can be shown that one impulse process should be I(1) and the other I(0); see, for example, Section 5.1 of Lau (1997). Assuming that the productivity and investment share impulses are I(1) without drift and I(0) respectively, then $c_P = 1$ and $c_I = 0$ in (3). In this case, the

⁴ The distinction of the two terms ‘external impulse’ ($\ln e_t^j$) and ‘structural disturbance’ (ε_t^j), which are related by (3), is maintained throughout the analysis. Specifically, while a structural disturbance process is restricted to be white noise, an external impulse process may be I(0) or I(1). Nevertheless, a structural disturbance may produce temporary or permanent effects, depending on other aspects of the growth model.

log-linearized equations of motion near the steady-state growth path have the following VMA form:

$$\begin{aligned} \begin{bmatrix} (1-L)\ln y_t \\ (1-L)\ln i_t \end{bmatrix} &= \begin{bmatrix} \tau \\ \tau \end{bmatrix} + \left(1 - \frac{(1+\tau) - (1-\alpha)(\tau+\delta)}{1+\tau}L\right)^{-1} \\ &\times \begin{bmatrix} \left(1 - \frac{1-\delta}{1+\tau}L\right)q_P^{-1}(L) & (1-L)\frac{\alpha(\tau+\delta)}{1+\tau}Lq_I^{-1}(L) \\ \left(1 - \frac{1-\delta}{1+\tau}L\right)q_P^{-1}(L) & (1-L)\left(1 - \frac{1-\delta}{1+\tau}L\right)q_I^{-1}(L) \end{bmatrix} \begin{bmatrix} \varepsilon_t^P \\ \varepsilon_t^I \end{bmatrix}. \end{aligned} \quad (4)$$

(All detailed derivations can be found in the Appendix.)

It can be observed that the long-run growth rate in this model is affected by the rate of technological progress only. The long-run effects on output and investment level of a disturbance to productivity and investment share, respectively, will be discussed later.

The simplest way to ‘endogenize’ the source of growth is the AK model (Rebelo, 1991). The (exogenous investment-share version of) stochastic AK model consists of (2), (3) and

$$Y_t = AK_t e_t^P. \quad (5)$$

It can be shown that if the order of integration of log per capita output or log per capita investment is at most one, then both external impulses should be $I(0)$ in this model, i.e., $c_P = c_I = 0$ in (3). See, for example, Section 5.2 of Lau (1997).

In this case, the log-linearized equations of motion near the steady-state growth path have the following VMA representation:

$$\begin{aligned} \begin{bmatrix} (1-L)\ln y_t \\ (1-L)\ln i_t \end{bmatrix} &= \begin{bmatrix} \ln(1-\delta+sA) \\ \ln(1-\delta+sA) \end{bmatrix} \\ &+ \begin{bmatrix} \left(1 - \frac{1-\delta}{1-\delta+sA}L\right)q_P^{-1}(L) & \frac{sA}{1-\delta+sA}Lq_I^{-1}(L) \\ \left(1 - \frac{1-\delta}{1-\delta+sA}L\right)q_P^{-1}(L) & \left(1 - \frac{1-\delta}{1-\delta+sA}L\right)q_I^{-1}(L) \end{bmatrix} \begin{bmatrix} \varepsilon_t^P \\ \varepsilon_t^I \end{bmatrix}. \end{aligned} \quad (6)$$

It is easy to observe from (6) that a permanent change in the investment share (represented by a change in parameter s) will produce growth effects on both output and investment.

The above analysis shows that both (4) of the Solow-Swan model and (6) of the AK model can be represented by the following first-difference VMA

form:⁵

$$\begin{bmatrix} \Delta x_{1t} \\ \Delta x_{2t} \end{bmatrix} = \text{constant} + M(L) \varepsilon_t \equiv \text{constant} + \begin{bmatrix} M_{11}(L) & M_{12}(L) \\ M_{21}(L) & M_{22}(L) \end{bmatrix} \varepsilon_t, \quad (7)$$

where $\Delta \equiv 1 - L$ is the first-difference operator, x_1 and x_2 stand for the log of per capita output and the log of per capita investment respectively, $\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t})'$ is a 2×1 vector of structural disturbances such that its components are serially and mutually uncorrelated, the 2×2 matrix of lag polynomial $M(L)$ is defined by $M(L) = \sum_{j=0}^{\infty} M_j L^j$, and the ‘constant’ vector is non-zero (in order for the variables to demonstrate long-run growth). How this ‘constant’ vector is related to the behavioral parameters is important to the long-run effects of permanent changes in these parameters, as illustrated in the above analysis and in Section 4 of Lau (1997). Other than this issue, its precise value is inconsequential to the analysis in this paper and therefore is left unspecified.

Following the terminology in the vector autoregression (VAR) literature, the time-invariant system (7) is called the structural model. Since these relationships are structural, the off-diagonal elements of the leading VMA parameter matrix M_0 may be non-zero; see, for example, (4) and (6). For a meaningful structural model, the inverse of M_0 exists. Note that the variance-covariance matrix of the structural disturbances ε_t is a diagonal matrix. Moreover, the two variances can be normalized to be one without loss of generality. The *long-run multiplier matrix* of the structural VMA representation (7) is given by

$$M(1) = \sum_{j=0}^{\infty} M_j, \quad (7a)$$

and its various elements contain information about the long-run effects of structural disturbances on the level of observed variables.

It can be observed from (4) that the long-run multiplier matrix of the stochastic Solow-Swan model is given by:

$$M(1) = \begin{bmatrix} \frac{\tau+\delta}{1+\tau} q_P^{-1}(1) & 0 \\ \frac{\tau+\delta}{1+\tau} q_P^{-1}(1) & 0 \end{bmatrix}. \quad (4a)$$

The first column of the long-run multiplier matrix in (4a) is non-zero and the second column is zero, reflecting that a productivity disturbance produces

⁵ More generally, the structural VMA representation of an exogenous growth model and an endogenous growth model are given respectively by (21) and (14) of Lau (1997). Both equations can be represented by a multivariate generalization of (7) of this paper.

permanent effects and a disturbance to investment share produces temporary effects. On the other hand, the long-run multiplier matrix of the stochastic AK model is given by:

$$M(1) = \begin{bmatrix} \frac{sA}{1-\delta+sA}q_P^{-1}(1) & \frac{sA}{1-\delta+sA}q_I^{-1}(1) \\ \frac{sA}{1-\delta+sA}q_P^{-1}(1) & \frac{sA}{1-\delta+sA}q_I^{-1}(1) \end{bmatrix}. \quad (6a)$$

Both columns of the long-run multiplier matrix in (6a) are non-zero, reflecting that the effect of either a disturbance to productivity or investment share is permanent. It can be observed from (4a) and (6a) that the 2×2 long-run multiplier matrix $M(1)$ of both models are of *reduced rank* of one, meaning that per capita output and per capita investment are *cointegrated*.

The above analysis shows that if the cointegrated system (7) is generated by an endogenous growth model, then both external impulses are $I(0)$ but each of them produces non-zero long-run effects and therefore both columns of $M(1)$ are non-zero; see (6) and (6a). Moreover, in a cointegrated system of per capita output and per capita investment, a positive disturbance to productivity (respectively investment share), which increases current per capita output (respectively per capita investment), is expected to produce positive (instead of negative) long-run effects on either variable for a meaningful endogenous growth model. Therefore, both columns of $M(1)$ are positive. On the other hand, if the cointegrated system (7) is generated by an exogenous growth model, then one external impulse process is $I(0)$ and the other is $I(1)$. As $I(0)$ impulses produce only temporary effects for exogenous growth model, one column of the long-run multiplier matrix $M(1)$ will be zero; see (4) and (4a).⁶

In summary, the long-run effects of permanent and temporary changes in economic fundamentals differ for endogenous and exogenous growth models. Moreover, the long-run effects of permanent and temporary changes are related as each is caused by the presence or absence of the factor $(1 - L)$ in the determinant of the polynomial matrix of the behavioral systems; see, for example, (37) and (32) of Lau (1997). The difference of the long-run effects of *permanent changes* in economic fundamentals between these two classes of models is a very sharp distinction: a permanent change in some behavioral parameters (such as the average investment share s) will lead to a growth effect for an endogenous growth model, whereas a permanent change in the underlying parameters, other than the exogenous rate of technological progress, leads only to a level effect for an exogenous growth model. Unfortunately, this

⁶ If both impulses are $I(1)$, then no column of $M(1)$ is zero and the system is not cointegrated. Therefore, the systematic difference between endogenous and exogenous growth models regarding the long-run effects of structural disturbances disappears for an $I(1)$ but non-cointegrated system.

implication may not be testable as it is difficult to find interesting events of permanent changes in behavioral parameters.

On the other hand, *temporary changes* in economic fundamentals are easier to find. Regarding the long-run effects of these temporary changes, there is also a systematic difference between these two classes of models *when the variables are cointegrated*. From the next section onwards, the difference of the long-run effects of temporary changes between endogenous and exogenous growth models will be exploited to develop a testing procedure, and the focus will be on the long-run multiplier matrix for the structural VMA representation (7) of a cointegrated system. As the above analysis makes clear, testing the long-run effect of a temporary change in economic fundamentals provides an indirect answer regarding the effect of a permanent change.

3 Investigating the long-run effect of a disturbance to investment share

The analysis of Section 2 shows that for a bivariate cointegrated system of per capita output and per capita investment, both columns of the long-run multiplier matrix $M(1)$ in (7) are positive if the data are generated by an endogenous growth model, but one column of $M(1)$ is positive and the other is zero if the data are generated by an exogenous growth model. Provided that the structural model is identified, this is a testable hypothesis.

This section elaborates the above idea and suggests a method to examine whether the long-run effect of an investment share disturbance on per capita output is zero or positive. Specifically, Sub-section 3.1 discusses how identification is achieved in this paper. Under the particular identifying assumption, Sub-section 3.2 then suggests a simple way to test the hypothesis that an investment share disturbance produces a zero long-run effect. Combining all the results, Sub-section 3.3 summarizes the empirical procedure used in this paper. Prior to these sub-sections, the next paragraph discusses the validity of the implicit assumption made in this paper that productivity disturbances always produce permanent effects.

When per capita output and per capita investment are cointegrated (and thus the rank of $M(1)$ in (7) is one), at least one structural disturbance always produces permanent effects. It is assumed in this paper that the productivity disturbance always produces a permanent effect but the investment share disturbance may produce a temporary or permanent effect. In the notation of this paper, the first column of $M(1)$ is non-zero but the second column may be zero or non-zero. This specification is justified on two grounds. First, such a specification is flexible to allow the data to decide whether a disturbance

to investment share produces a temporary effect or not, which is the main objective of this paper. Changing the roles of the two structural disturbances just presupposes the answer. Second, if the investment share disturbance is interpreted according to (2) and (3), then the above assumption regarding the roles of the two structural disturbances is equivalent to assuming that the external impulse to the investment share is $I(0)$. In this case, $c_I = 0$ in (3) and the investment-output ratio is stationary, which is consistent with the empirical evidence on industrial countries; see, for example, King et al. (1991).

3.1 Identifying assumptions

Regarding the assumptions used to identify the structural model, this paper makes use of the idea in the theoretical growth literature that *current output is determined in the supply side* by factor inputs such as capital and labor. Specifically, it is assumed in many growth models that current output is determined in the supply side according to the production function, as illustrated in (1) of the Solow-Swan model or (5) of the AK model. On the other hand, the decision between current consumption and investment for future capital is made on the basis of some demand-side or preference factors, such as the simple investment rule used in the Solow-Swan model or a more complicated mechanism based on optimizing an intertemporal objective function. As a result, variables (such as investment) which are not present in the production function do not affect current output but may still affect future output through, for example, capital accumulation.

When the idea of supply-side determination of current output is interpreted in a stochastic setting, it implies a *recursive ordering* regarding the structural disturbances. Only disturbances affecting the production technology (ε_t^P in the models of Section 2) or the inputs will affect output contemporaneously, whereas other variables such as investment are contemporaneously affected by all disturbances. (The assumptions used in Kocherlakota and Yi (1996, footnote 4) also reflect the supply-side determination of current output.) For the bivariate system studied in this paper, the disturbance to investment share does not affect per capita output contemporaneously (i.e., $m_{0,12} = 0$ in the notation of this paper, where $m_{0,ij}$ is the row- i , column- j element of the leading VMA parameter matrix M_0); see (4) or (6). As a result, the use of Sims' (1980) recursive identification scheme (with per capita output ordered first) is valid when current output is determined in the supply side.⁷

⁷ It should be emphasized that this recursive ordering depends only on the assumption that current output is determined in the supply side, and therefore may be present in an optimizing model or in an exogenous investment share model. To make the analysis relatively simple and intuitive, Section 2 uses an exogenous investment share assumption, and thus the structural disturbance ε^I in (2) is natu-

Of course, no identifying assumption is appropriate in all circumstances or is consistent with all theoretical models. The above identification scheme is chosen as it reflects the maintained hypothesis that the data are generated by some growth models. Moreover, this choice is neutral with respect to the exogenous versus endogenous growth debate.

3.2 A test of zero long-run effect and the underlying idea

Under the assumption that a disturbance to productivity will always produce permanent effects, the analysis in Section 2 shows that the long-run effect of a disturbance to investment share is zero (i.e., $M_{12}(1) = M_{22}(1) = 0$) for an exogenous growth model, but is positive for an endogenous growth model. Thus, the rejection of zero long-run effect of an investment share disturbance is a necessary (but not sufficient) condition for the cointegrated system (7) to be generated by an endogenous growth model. This sub-section shows that under the identifying assumption of $m_{0,12} = 0$, there is a simple way to test the null hypothesis of $M_{12}(1) = M_{22}(1) = 0$ (i.e., the second column of $M(1)$ is zero). The test is based on the estimated coefficients of the following reduced-form bivariate vector error-correction system:⁸

$$\begin{aligned} \begin{bmatrix} \Delta x_{1t} \\ \Delta x_{2t} \end{bmatrix} &\equiv \begin{bmatrix} \beta_1^R \\ \beta_2^R \end{bmatrix} + \sum_{j=1}^p \begin{pmatrix} \pi_{j,11}^R & \pi_{j,12}^R \\ \pi_{j,21}^R & \pi_{j,22}^R \end{pmatrix} \begin{bmatrix} \Delta x_{1,t-j} \\ \Delta x_{2,t-j} \end{bmatrix} \\ &+ \begin{bmatrix} \lambda_1^R \\ \lambda_2^R \end{bmatrix} (x_{2,t-1} - x_{1,t-1}) + \begin{bmatrix} \varepsilon_{1t}^R \\ \varepsilon_{2t}^R \end{bmatrix}, \end{aligned} \quad (8)$$

rally interpreted as a disturbance to investment share. Alternatively, if the bivariate system of per capita output and per capita investment is generated by a more elaborate model (such as an optimizing model with government), then the structural disturbance ε^I may be interpreted as a mixture of disturbances to government policy and the agent's utility function. For example, a negative disturbance to ε^I in the bivariate framework of this paper may capture a positive disturbance to income tax revenue (the type of disturbances considered in Stokey and Rebelo, 1995).

⁸ In studying a bivariate system, it is implicitly assumed that the specification of only two structural disturbances is meaningful; see, for example, Cochrane (1994), Lastrapes and Selgin (1994), Quah and Vahey (1995) and, especially, Blanchard and Quah (1989). When the number of underlying disturbances is more than two, the proposed test in a bivariate cointegrated system to distinguish between the endogenous and exogenous growth models is still valid under the assumptions that (a) current output is determined in the supply side and (b) the 'other disturbance' (ε^I), which captures all disturbances orthogonal to the output disturbance and is identified as having no contemporaneous effect on output, produces a zero long-run effect on output for an exogenous growth model.

where p is the lag length, and $\lambda^R = (\lambda_1^R, \lambda_2^R)'$ is the reduced-form adjustment vector. Note that the components of $\varepsilon_t^R = (\varepsilon_{1t}^R, \varepsilon_{2t}^R)'$ in this reduced-form system are serially uncorrelated (which is a reasonable assumption provided that there are enough lag terms) but may be contemporaneously correlated with each other.

Under the identifying assumption of $m_{0,12} = 0$, it can be shown that testing whether λ_1^R , the first element of the adjustment vector of the reduced-form vector error-correction model (8), is zero or non-zero can be used to examine whether $M_{12}(1) = M_{22}(1) = 0$ or not.⁹ More generally, the above result can be stated as:

Proposition 1. *If Sims' x_1 -first ordering is adopted for the bivariate cointegrated system of variables x_1 and x_2 , then testing whether λ_1^R in (8) is zero or not can be used to examine whether the long-run effect of a structural x_2 -disturbance is zero or not. By symmetry, if Sims' x_2 -first ordering is adopted, then testing whether λ_2^R is zero or not can be used to examine whether the long-run effect of a x_1 -disturbance is zero or not.*

The proof is given in the Appendix.¹⁰ A brief discussion of the underlying idea of Proposition 1 is given as follows. There is an assumption regarding the statistical property of the data (that the two variables are cointegrated), and there is an identifying assumption regarding the structural disturbances (that there is no contemporaneous effect of a x_2 -disturbance on the other variable x_1).

Since the rank of the long-run multiplier matrix for the structural VMA representation (7) of a bivariate cointegrated system is one, at least one column of the matrix must be non-zero. Through the relationships among various representations of a cointegrated system (see, for example, Engle and Granger, 1987, and Banerjee et al., 1993), it can be shown that the null hypothesis of $M_{12}(1) = M_{22}(1) = 0$ (which corresponds to a stochastic exogenous growth model in the context of the endogenous versus exogenous growth debate) implies that λ_1 is zero, where λ_1 is the coefficient of the error-correction term in the first equation of the following structural vector error-correction model

⁹ Note that the test of $\lambda_1^R = 0$ is the same as testing for weak exogeneity of variable x_1 under the maintained assumption of cointegration in (8); see, for example, Johansen (1995, Theorem 8.1). I am grateful to a referee for making this point.

¹⁰ Proposition 1, which holds for a bivariate cointegrated system, can be generalized to multivariate cointegrated systems; see the Appendix. However, identifying assumptions in addition to the one based on growth theory (that current output is determined in the supply side) are required if a higher dimensional system is studied; see footnote 21. Thus, I only focus on a bivariate system in the empirical analysis.

corresponding to (8):

$$\begin{aligned} \Pi_0 \begin{bmatrix} \Delta x_{1t} \\ \Delta x_{2t} \end{bmatrix} &\equiv \begin{bmatrix} \pi_{0,11} & \pi_{0,12} \\ \pi_{0,21} & \pi_{0,22} \end{bmatrix} \begin{bmatrix} \Delta x_{1t} \\ \Delta x_{2t} \end{bmatrix} = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} \\ + \sum_{j=1}^p \begin{bmatrix} \pi_{j,11} & \pi_{j,12} \\ \pi_{j,21} & \pi_{j,22} \end{bmatrix} \begin{bmatrix} \Delta x_{1,t-j} \\ \Delta x_{2,t-j} \end{bmatrix} &+ \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} (x_{2,t-1} - x_{1,t-1}) + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}. \end{aligned} \quad (9)$$

Finally, the relationship between the structural model (9) and the reduced-form model (8) implies the following result:¹¹

$$\begin{bmatrix} \lambda_1^R \\ \lambda_2^R \end{bmatrix} \equiv \lambda^R = M_0 \lambda = \begin{bmatrix} m_{0,11} \lambda_1 + m_{0,12} \lambda_2 \\ m_{0,21} \lambda_1 + m_{0,22} \lambda_2 \end{bmatrix}. \quad (10)$$

Combining the identifying assumption of $m_{0,12} = 0$ with the hypothesis of $\lambda_1 = 0$ about the unobserved structural model leads to a testable implication of $\lambda_1^R = 0$ about the reduced-form model, according to (10).

3.3 The empirical procedure

The above results suggest the following procedure to examine the long-run effect of an investment share disturbance. The first step is to estimate the reduced-form vector error-correction model (8). Based on the estimated coefficients (as well as the estimated variance-covariance matrix), the next step is to test $\lambda_1^R = 0$ and to obtain the long-run response of an investment share disturbance on per capita output, M_{12} (1), under the identifying assumption $m_{0,12} = 0$.¹² As mentioned above, the test of $\lambda_1^R = 0$ is a test of the hypothesis

¹¹ Premultiplying the structural vector error-correction model (9) by $(\Pi_0)^{-1}$ and comparing it with its reduced-form counterpart (8) give (10) and $\varepsilon_t^R = (\Pi_0)^{-1} \varepsilon_t = M_0 \varepsilon_t$, where $M_0 = (\Pi_0)^{-1}$ can be shown to hold. See, for example, Cochrane (1994). Moreover, the above relationship and the normalization of variances of the structural disturbances imply that the reduced-form variance-covariance matrix, $E(\varepsilon_t^R \varepsilon_t^{R'})$, is related to M_0 according to $E(\varepsilon_t^R \varepsilon_t^{R'}) = M_0 E(\varepsilon_t \varepsilon_t') M_0' = M_0 M_0'$. For a bivariate system, this relationship gives three independent restrictions. With one more restriction, the four elements of M_0 can be recovered and thus the structural model is identified.

¹² The long-run responses can be obtained by standard methods for impulse-response function. Since the distinguishing feature between endogenous and exogenous growth models lie on the long-run response, the impulse responses at other lags are not presented in this paper.

of zero long-run effect of a structural disturbance to investment share, under the identifying assumption $m_{0,12} = 0$.

If the null hypothesis $\lambda_1^R = 0$ is rejected and the estimated long-run response of an investment share disturbance on per capita output, $M_{12}(1)$, is positive, then the evidence is more favorable to an endogenous growth model. On the other hand, if either (a) the null hypothesis $\lambda_1^R = 0$ is not rejected, or (b) the null hypothesis $\lambda_1^R = 0$ is rejected but the estimated long-run response $M_{12}(1)$ is negative, then the evidence is less favorable to an endogenous growth model.

4 Empirical analysis

The empirical analysis focuses on six major industrial countries—Canada, France, Italy, Japan, UK and USA—in the post World War II period.¹³ The data are taken from the Penn World Table version 6.1 (PWT6.1); a description of an earlier version 5 is given in Summers and Heston (1991). The per capita real GDP figures are the ‘RGDPL’ series, and the per capita real investment figures are obtained from combining the ‘RGDPL’ and ‘KI’ series (real gross domestic investment as percentage of real GDP). The various series for these four countries are plotted in Figures 1 to 6. All the (annual) series are from 1950 to 2000, and are measured in 1996 international prices. The post-war period is chosen primarily because of compatibility with previous studies on the effect of investment share on output, such as of Levine and Renelt (1992) and, especially, Jones (1995, Section III). The former study uses cross-country data from 1960 to 1989, and the latter uses annual series of industrial countries from 1950 to 1988 and mentions that the choice of that sample avoids changes in the stochastic properties associated with World War II (p. 501).

[Insert Figures 1 to 6 here.]

4.1 Estimation and testing

The maximum order of the vector error-correction model is assumed to be four. The Akaike Information Criterion (AIC), the Bayesian Information Criterion (BIC), and the Durbin-Watson statistics for the models with various lags are presented in Table 1. For Canada and France, the model with one lag term is chosen by both the AIC and BIC. For Italy, Japan, UK and USA, the model

¹³The real GDP per worker series for Germany—the remaining member of the Group of Seven (G7) industrial countries—starts from 1990 and other variables start from 1970 in the PWT6.1 data set. As these series are too short for meaningful analysis, Germany is not examined in this paper.

with two lag terms is chosen by the AIC and the model with one lag is chosen by the BIC. Based on the Durbin-Watson statistics, as well as the correlograms and the Ljung-Box Q -statistics (in Figures A1 to A6), the model with two lags is the more preferred choice for each of these countries. To conserve space, only the results for the model with two lags are focused for these four countries, but the robustness of the conclusion with respect to the number of lag terms will be examined. The various regression equations run from 1955 to 2000, with the earlier observations used as starting values.

[Insert Table 1 here.]

The empirical procedure of this paper is developed for a bivariate cointegrated system. To test whether or not log per capita output and log per capita investment are cointegrated with a prespecified cointegrating vector $(1, -1)$, this paper uses the cointegration test suggested in Horvath and Watson (1995), which has been shown to possess better power properties when compared to other tests that do not impose the cointegrating vector. The Horvath-Watson test examines the null hypothesis of no cointegration against the composite alternative of cointegration with a known cointegrating vector. Moreover, this test fits nicely into the error-correction model (8). In this case, the null hypothesis of no cointegration corresponds to $\lambda_1^R = \lambda_2^R = 0$ in (8). To test this hypothesis, a likelihood ratio test is performed by calculating the log-likelihood of the unrestricted model (8), as well as the log-likelihood of the restricted model:

$$\begin{bmatrix} \Delta x_{1t} \\ \Delta x_{2t} \end{bmatrix} \equiv \begin{bmatrix} \beta_1^R \\ \beta_2^R \end{bmatrix} + \sum_{j=1}^p \begin{pmatrix} \pi_{j,11}^R & \pi_{j,12}^R \\ \pi_{j,21}^R & \pi_{j,22}^R \end{pmatrix} \begin{bmatrix} \Delta x_{1,t-j} \\ \Delta x_{2,t-j} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t}^R \\ \varepsilon_{2t}^R \end{bmatrix}. \quad (11)$$

The test statistic is given by:

$$LR = 2(\ln L_u - \ln L_R), \quad (12)$$

where $\ln L_u$ is the log-likelihood of the unrestricted model (8) and $\ln L_R$ is the log-likelihood of the restricted model (11). The critical values of the test statistic are given in Table 1 of Horvath and Watson (1995).

The results of the cointegration test are presented in Table 2. It can be observed that the null hypothesis is rejected at a 1% significance level for France, Italy and Japan, is rejected at a 10% significance level for UK, but is not rejected at a 10% significance level for Canada and USA. For France, Italy, Japan and UK, there is evidence that the log of investment-output ratio is $I(0)$. As a result, the remaining empirical procedure of this paper is applied to these four countries but not to Canada and USA; see footnote 6 as well.

[Insert Table 2 here.]

The regression results of the vector error-correction model (8) for France, Italy, Japan and UK, with the lag length in each country determined by AIC, are presented in Table 3. Based on the estimated coefficients and the identifying assumption that a disturbance to investment share does not affect current per capita output, the long-run responses of the level of per capita output to both structural disturbances can be obtained. They are also given in Table 3.

[Insert Table 3 here.]

According to Proposition 1, the coefficient of the error-correction term for the per capita output equation can be used to test whether the long-run effect of an investment share disturbance is zero or not if this disturbance does not affect per capita output contemporaneously.¹⁴ It is observed from Table 3 that the t -statistic of the estimated coefficient of the error-correction term of the per capita output equation is 0.49 for Italy and -0.95 for UK. Thus, the coefficient λ_1^R is not statistically different from zero at a 5% significance level. On the other hand, the t -statistic of the estimated coefficient of the error-correction term of the per capita output equation is -2.79 for France and -3.42 for Japan, but the estimated long-run response, $M_{12}(1)$, is negative in each case (-0.014 for France and -0.035 for Japan). Thus, the null hypothesis that the data are generated by an exogenous growth model is not rejected by the post-war data of each of the four G7 countries in which the cointegrated system (8) is appropriate.

4.2 Robustness analysis

A practical virtue of the simple statistical procedure (OLS regression) used in the above analysis is that the results are quite robust with respect to the number of lag terms included. First of all, if the number of lag terms is chosen by the BIC, then the likelihood ratio statistics of the Horvath and Watson (1995) test are given in the upper panel of Table 4. It can be seen that the null hypothesis that the log of investment share is $I(0)$ can be rejected at a 5% significance level for UK, and at a 1% significance level for Italy and Japan,

¹⁴In the presence of non-stationary time series, a natural question is whether the distributions of the various test statistics (especially those of the error-correction coefficients) are standard or not under the null hypothesis. The results in Sims et al. (1990) make clear that under the maintained assumption that the variables are cointegrated, the t -statistic for the null hypothesis follows standard distribution asymptotically because all the right-hand side variables of the regressions are $I(0)$. Note that the test $\lambda_1^R = 0$ is a test of zero long-run effect under the maintained assumption of cointegration, and is neither a unit root nor cointegration test. On the other hand, the asymptotic distribution of the null hypothesis $\lambda_1^R = \lambda_2^R = 0$ (i.e., the null hypothesis of no cointegration) is non-standard; see Table 2.

but is not rejected at a 10% significance level for USA. These conclusions are essentially the same whether the number of lag terms are determined by the AIC or BIC.

[Insert Table 4 here.]

The estimated coefficients and the t -statistics for the error-correction terms, and the long-run responses of the structural disturbances (under the recursive ordering with per capita output first) for each country with various lags are presented in the upper panel of Table 5. It can be observed that the estimated coefficients of the error-correction terms and the associated t -statistics for the models with different lags remain relatively unchanged in each country.¹⁵ For the various models with one to four lags in either France, Italy, Japan or UK, it is still concluded that a disturbance to investment share does not have a positive long-run effect on per capita output.

[Insert Table 5 here.]

In the theoretical models in Section 2, the simplifying assumption of inelastic labor supply is used, as in most growth models. In empirical papers examining the relevance of theoretical growth models, researchers have used either the data based on total population or total number of workers. To see whether the above conclusion is robust with respect to the choice of variables, this paper also uses the real GDP per worker series ('RGDPWOK' series in PWT61) and the real investment per worker series (by combining the 'RGDPWOK' and 'KI' series).

In the lower panel of Table 4, the likelihood ratio statistics are presented. It can be observed that the null hypothesis that log output per worker and log investment per worker are $I(1)$ but not cointegrated is rejected at a 10% significance level for UK, at a 5% significance level for France and at a 1% significance level for Italy and Japan, but is not rejected at a 10% significance level for Canada and USA.

In the lower panel of Table 5, it is observed that the t -statistic of the estimated coefficient of the error-correction term of the output per worker equation is 0.07 for Italy and -1.21 for UK. Thus, the coefficient λ_1^R is not statistically different from zero at a 5% significance level. On the other hand, the t -statistic of the estimated coefficient of the error-correction term of the output per worker equation is -3.17 for France and -3.52 for Japan, but the estimated long-run response, $M_{12}(1)$, is negative in each case (-0.022 for France and -0.036 for Japan). Again, the null hypothesis that the data, based on total number of

¹⁵ According to unreported results, $M_{11}(1)$ is positive in all cases. Thus, the implicit assumption that the productivity disturbance produces positive long-run effects is satisfied.

workers, are generated by an exogenous growth model is not rejected by the post-war data of France, Italy, Japan and UK.

4.3 *Parameter constancy*

The data in the above analysis cover the second half of the twentieth century. During this period (especially the seventies and eighties), a number of major events happened, including the collapse of the Bretton Woods system, oil crises, rising inflation and the subsequent disinflation. A natural question is whether the bivariate error-correction models of the various industrial countries remain stable during the sample period.

To examine whether the estimated coefficients of the various models in Table 3 are stable during this period, the CUSUM plots (Brown et al., 1975) of the various equations in Table 3 are given in Figures 7 to 10.¹⁶ Since the long-run effects of investment share disturbances are obtained from these estimated coefficients, the conclusions would be less convincing if the CUSUM tests indicate instability.

[Insert Figures 7 to 10 here.]

For France and Japan, the CUSUM plots for the output and investment equations stay inside the 95% confidence lines and show no evidence of parameter instability. For UK, the CUSUM plots indicate the stability of the output equation, but the investment equation is on the borderline of the 95% band in the early eighties before moving inside the band again. For Italy, there is evidence of instability for both equations. Unreported recursive estimates indicate that abrupt changes occur for the output and investment equations of the three European countries, but the timing of the changes differ—France in the mid-seventies, Italy in late seventies, and the UK in early eighties. The magnitude of the abrupt change is relatively mild for France and UK, but is stronger for Italy causing the CUSUM statistics to move outside the 95% confidence lines.¹⁷ On the other hand, the recursive estimates of output and investment equations of Japan are rather stable over time, except for two parameters in

¹⁶ The CUSUM test is first developed in a regression model when all regressors are independent of the disturbances. Krämer et al. (1988) show that the test is asymptotically valid even if there are lagged dependent variables.

¹⁷ While an in-depth examination of the reasons of the parameter instability for Italy is beyond the scope of this paper, it is noted that the inflation rates of Italy from the mid-seventies to mid-eighties were abnormally high (staying higher than 15% for most of these years). Italy also suffered from a deep recession (with a real growth rate of -4%) in 1975. See, for example, Chart 2 of Vercelli and Fiordoni (2003).

the late eighties.

Based on the CUSUM tests and the recursive estimates, it can be concluded that the bivariate cointegrated system (8) is reasonably stable during the sample period for France, Japan and UK, but not for Italy.

4.4 Summary

To summarize, when per capita output and per capita investment are cointegrated and Sims' output-first ordering is valid, the test of $\lambda_1^R = 0$ in (8) and the sign of $M_{12}(1)$ can be used to examine whether a disturbance to investment share produces a zero or positive effect on per capita output. For France, Japan and UK, it is concluded that a temporary change in investment share does not produce a positive long-run effect on per capita output. (This conclusion also holds for Italy, but there is evidence of coefficient instability over time.) The evidence is more consistent with the class of exogenous growth models than with the class of endogenous growth models. This conclusion holds whether per capita series or per worker series are used, and is also robust to the number of lag terms included.

5 Conclusion

A major empirical interest, and a distinguishing characteristic between endogenous and exogenous growth models, is whether a permanent change in investment share produces a growth effect or only a level effect. Unfortunately, a direct time series analysis of this hypothesis may not always be possible due to a lack of permanent changes in investment share. In fact, for most countries such as France, Italy, Japan and UK, investment-output ratios fluctuate around a constant mean and there is no evidence of a permanent change in this variable; see the discussion in King et al. (1991) and Cochrane (1994), as well as Figures 2 to 5 of this paper. To prevent determining the empirical relevance of the endogenous growth models by examining the long-run effects of a (possibly non-existent) *permanent change* in investment share, this paper takes a complementary approach. It assumes that such a permanent change is absent (or at least cannot be established affirmatively) in the data, and considers the long-run effect of a *temporary change* in investment share.

The first contribution (Section 2) of this paper is to relate the apparently distinct questions of the long-run effect of a permanent change in economic fundamentals (as examined in Jones, 1995; Stokey and Rebelo, 1995) and that of a temporary change (as examined in Kocherlakota and Yi (1996) and this

paper). It then suggests that when log per capita output and log per capita investment are cointegrated, testing the long-run effect of a temporary change in investment share on per capita output provides an indirect answer regarding the growth versus level effect debate.

The second contribution (Section 3) of the paper is to develop a method to examine whether the long-run effect of a disturbance to investment share on per capita output is zero or positive. Using the systematic difference in time series properties of endogenous and exogenous growth models (in Section 2) and the relationships of different representations of a cointegrated system (in Section 3), this paper suggests a simple test of zero long-run effect of a disturbance to investment share. Specifically, under the assumption that an investment share disturbance does not affect per capita output contemporaneously (which is consistent with the idea commonly found in the growth literature that current output is determined in the supply side), Proposition 1 shows that the coefficient of the error-correction term for the per capita output equation in model (8) can be used to test whether the long-run effect of a temporary change in investment share is significantly different from zero.

The empirical framework of this paper shares some similarities with Kocherlakota and Yi (1996), as the long-run effects of temporary changes in economic fundamentals are examined in both papers. However, there are also a number of differences between the two papers. Kocherlakota and Yi (1996) focus on a set of policy variables for USA, while investment shares of several industrial countries are examined in this paper. Kocherlakota and Yi (1996, p. 127) provide statistical justifications for the three cases in which their methods are appropriate, whereas this paper combines economic analysis and time series properties of the data to suggest a method to distinguish between endogenous and exogenous growth models when per capita output and per capita investment are cointegrated. Finally, Kocherlakota and Yi (1996) use a single-equation approach, but this paper uses a system approach.¹⁸

The third contribution (Section 4) of this paper is about empirical evidence. It is found that a disturbance to investment share does not have a positive long-run effect on per capita output for France, Japan and the UK (and Italy too, but there is evidence of parameter instability for Italy). There is no evidence favorable to the endogenous growth models. These results are quite robust with respect to lag length and to the choice of variables. Even though this

¹⁸ While only λ_1^R of the per capita output equation is required to test the null hypothesis of zero long-run effect of an investment share disturbance under the assumption that an investment share disturbance does not affect per capita output contemporaneously (and therefore, Sims' X_1 -first ordering is valid), both equations of (8) are required to estimate the long-run response M_{12} (1). Moreover, the evidence based on both equations is useful in determining the lag length according to the AIC or BIC.

paper assumes that the investment-output ratio is stationary and therefore does not share with a major premise in Jones (1995) regarding the presence of deterministic or stochastic trend in this ratio, the conclusions obtained in these two papers are similarly unfavorable to endogenous growth models.¹⁹ Moreover, they are complementary in the sense that no positive growth effect associated with a permanent change in investment share is found in one study and no positive permanent level effect associated with a temporary change in investment share is found in the other. The time series evidence appears to be quite different from the positive and significant relationship between investment share and output growth found in cross-section studies.

With the focus on examining whether the long-run effect of a temporary change in investment share on per capita output is zero or positive, the objectives of this paper are to derive testable implications of some theoretical growth models and then to perform empirical analysis for post-war data of the major industrial countries. Other interesting questions are not investigated in this paper and left to future research. One possible extension is to examine the above hypothesis by combining the information of France, Japan and UK (and possibly other industrial countries as well, provided that the stationarity assumption for investment share is reasonable). While this could simply be done by testing the joint hypothesis that the coefficients of the error-correction terms for the per capita output equations in all three countries are zero, it may be better to first derive and then test the implications of theoretical growth models allowing for capital and/or labor mobility. It is interesting to see whether the empirical evidence based on testing the implications of growth models with factor mobility is consistent with those examining growth models in which each economy is implicitly assumed to evolve independently of others. Another, and a more ambitious, task would be to reconcile the different methodologies and conclusions used in cross-section studies (such as Barro, 1991; Mankiw et al., 1992; Levine and Renelt, 1992) and time series studies (such as Jones (1995) and this paper) regarding the relationship between investment share and per capita output growth. This investigation would potentially parallel the interesting work of Bernard and Durlauf (1996) which provides a unified framework for interpreting the diverse definitions, methods and results of cross-section and time series tests of the convergence hypothesis.

¹⁹ The conclusions of these two papers using time series analysis are not necessarily inconsistent with the evidence of Kocherlakota and Yi (1996)—another time series study which, however, does not examine the relationship between investment share and output growth—even though they have found some evidence in favor of endogenous growth models. They examine US data starting from 1910s for taxes, tariffs, money growth rate, government equipment and structural capital (both military and non-military) and find robust evidence supporting endogenous growth models based on government non-military structural capital but not the other six variables.

6 Appendix 1: Detailed derivations about the stochastic Solow-Swan model

Capital stock evolves according to:

$$K_{t+1} = I_t + (1 - \delta) K_t. \quad (\text{A1})$$

For the stochastic Solow-Swan model, denote in lower case letter with a circumflex ($\hat{\cdot}$) the variable per efficiency unit (i.e., the variable per capita divided by $(1 + \tau)^t$). The production function (1) can be represented as:

$$\hat{y}_t = A \hat{k}_t^\alpha e_t^P. \quad (\text{A2})$$

Equations (2), (A1) and (A2) lead to:

$$(1 + \tau) \left(\frac{\hat{y}_{t+1}}{A e_{t+1}^P} \right)^\frac{1}{\alpha} = (1 - \delta) \left(\frac{\hat{y}_t}{A e_t^P} \right)^\frac{1}{\alpha} + s \hat{y}_t e_t^I. \quad (\text{A3})$$

It is easy to show that the steady state with $\ln e^P$ and $\ln e^I$ at their mean value of zero is given by:

$$\hat{y} = \left(\frac{s A^\frac{1}{\alpha}}{\tau + \delta} \right)^\frac{\alpha}{1-\alpha}. \quad (\text{A4})$$

Combining (2) and the log-linearization of (A3) around the steady state path gives the dynamic system:

$$\begin{bmatrix} 1 - \frac{1-\delta}{1+\tau}L - \frac{\alpha(\tau+\delta)}{1+\tau}L \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \ln y_t - \tau t \\ \ln i_t - \tau t \end{bmatrix} = \begin{bmatrix} \left(\frac{\tau+\delta}{1+\tau} \right) \ln \left[\frac{A}{(\tau+\delta)^\alpha} \right] \\ \ln s \end{bmatrix} + \begin{bmatrix} \left(1 - \frac{1-\delta}{1+\tau}L \right) \ln e_t^P \\ \ln e_t^I \end{bmatrix}. \quad (\text{A5})$$

Solving the two equations in (A5) simultaneously leads to the univariate representation of per capita output and per capita investment as:

$$\begin{aligned} & \left(1 - \frac{(1 + \tau) - (1 - \alpha)(\tau + \delta)}{1 + \tau} L \right) (\ln y_t - \tau t) \\ &= \left(\frac{\tau + \delta}{1 + \tau} \right) \ln \left[\frac{s^\alpha A}{(\tau + \delta)^\alpha} \right] + \left(1 - \frac{1 - \delta}{1 + \tau} L \right) \ln e_t^P + \frac{\alpha(\tau + \delta)}{1 + \tau} L \ln e_t^I, \end{aligned} \quad (\text{A6})$$

and

$$\begin{aligned} & \left(1 - \frac{(1 + \tau) - (1 - \alpha)(\tau + \delta)}{1 + \tau} L \right) (\ln i_t - \tau t) \\ &= \left(\frac{\tau + \delta}{1 + \tau} \right) \ln \left[\frac{s A}{(\tau + \delta)^\alpha} \right] + \left(1 - \frac{1 - \delta}{1 + \tau} L \right) \ln e_t^P + \left(1 - \frac{1 - \delta}{1 + \tau} L \right) \ln e_t^I. \end{aligned} \quad (\text{A7})$$

When the productivity and investment share impulses are I(1) without drift and I(0) respectively, then $c_P = 1$ and $c_I = 0$ in (3). Combining (3) with the first difference of (A6) and (A7) give (4) of the main text.

7 Appendix 2: Detailed derivations about the stochastic AK model

For the AK model, (2), (5) and (A1) lead to:

$$y_{t+1}e_t^P = Ai_t e_t^P e_{t+1}^P + (1 - \delta) y_t e_{t+1}^P. \quad (\text{A8})$$

Represent the steady state growth path with $\ln e^P$ and $\ln e^I$ at their mean value of zero by $(\{y_t^*\}, \{i_t^*\}, \{c_t^*\}, \{k_t^*\})$. It can be shown that:

$$y_{t+1}^* = (1 - \delta + sA) y_t^*. \quad (\text{A9})$$

It is easy to observe that the variables grow at the same rate $sA - \delta$ (which is assumed to be positive) at the steady state. Combining (2) and the log-linearization of (A8) around the steady state growth path gives the dynamic system:

$$\begin{aligned} & \begin{bmatrix} 1 - \frac{1-\delta}{1-\delta+sA}L & -\frac{sA}{1-\delta+sA}L \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \ln y_t \\ \ln i_t \end{bmatrix} \\ = & \begin{bmatrix} \ln(1 - \delta + sA) - \frac{sA}{1-\delta+sA} \ln s \\ \ln s \end{bmatrix} + \begin{bmatrix} \left(1 - \frac{1-\delta}{1-\delta+sA}L\right) \ln e_t^P \\ \ln e_t^I \end{bmatrix}. \end{aligned} \quad (\text{A10})$$

Solving the two equations in (A10) simultaneously leads to the univariate representation of per capita output and per capita investment as:

$$(1 - L) \ln y_t = \ln(1 - \delta + sA) + \left(1 - \frac{1 - \delta}{1 - \delta + sA}L\right) \ln e_t^P + \frac{sA}{1 - \delta + sA}L \ln e_t^I, \quad (\text{A11})$$

and

$$(1 - L) \ln i_t = \ln(1 - \delta + sA) + \left(1 - \frac{1 - \delta}{1 - \delta + sA}L\right) \ln e_t^P + \left(1 - \frac{1 - \delta}{1 - \delta + sA}L\right) \ln e_t^I. \quad (\text{A12})$$

Combining $c_P = c_I = 0$ in (3) with (A11) and (A12) give (6) of the main text.

8 Appendix 3: Proof of Proposition 1

This Appendix presents the proof of Proposition 1. In fact, it presents a more general proof which is applicable for a cointegrated system with n ($n \geq 2$) variables. The proof is broken down into several components so that the underlying logic can be seen more clearly.

Let X_t is a $n \times 1$ vector of $I(1)$ variables with r ($1 \leq r \leq n - 1$) cointegrating

vectors. Its reduced-form VMA representation is given by:

$$(1 - L) X_t = \text{constant} + M^R(L) \varepsilon_t^R, \quad (\text{A13})$$

where ε_t^R is a $n \times 1$ vector of reduced-form disturbances, and $M^R(L)$ is a $n \times n$ matrix of lag polynomial with $M^R(0) = I_n$.

According to the Granger Representation Theorem (Engle and Granger, 1987; Banerjee et al., 1993, Section 5.3), we know that²⁰

(a)

$$\text{rank} [M^R(1)] = n - r. \quad (\text{A14})$$

(b) There exists a reduced-form vector error-correction representation:

$$\Pi^R(L) (1 - L) X_t = \text{constant} + \lambda^R \beta' X_{t-1} + \varepsilon_t^R, \quad (\text{A15})$$

where $\Pi^R(L)$ is related to $M^R(L)$, $\Pi^R(0) = I_n$, and β (the cointegrating matrix) and λ^R (the reduced-form adjustment matrix) are $n \times r$ matrices of rank r .

(c)

$$M^R(1) \lambda^R = 0_{n \times r}. \quad (\text{A16})$$

Now, consider the structural model of this system. Its VMA representation is given by:

$$(1 - L) X_t = \text{constant} + M(L) \varepsilon_t, \quad (\text{A17})$$

where ε_t is a $n \times 1$ vector of structural disturbances, and $M(L)$ is a $n \times n$ matrix of lag polynomial. The long-run multiplier matrix of the structural VMA representation is given by $M(1)$. Note that (7) with $n = 2$ is a special case of (A17).

The above results lead to the following Lemma.

Lemma A-1.

(a)

$$\text{rank} [M(1)] = n - r. \quad (\text{A18})$$

(b) There exists a structural vector error-correction representation

$$\Pi(L) (1 - L) X_t = \text{constant} + \lambda \beta' X_{t-1} + \varepsilon_t, \quad (\text{A19})$$

²⁰Note that there are other results in the Granger Representation Theorem, but they are not stated here as they are less relevant to the issues studied in this paper.

where $\Pi(L)$ is related to $M(L)$, and β and λ (the structural adjustment matrix) are $n \times r$ matrices of rank r .

(c) The structural long-run multiplier matrix in (A17) and the structural adjustment matrix in (A19) are related by:

$$M(1)\lambda = 0_{n \times r}. \quad (\text{A20})$$

Proof. First, by comparing (A13) with $M^R(0) = I_n$ and (A17), and using

$$M(L)\varepsilon_t = M(L)M^{-1}(0)M(0)\varepsilon_t,$$

one obtains the following relationships:

$$M^R(L) = M(L)M^{-1}(0), \quad (\text{A21})$$

and

$$\varepsilon_t^R = M(0)\varepsilon_t. \quad (\text{A22})$$

Since the matrix $M(0)$ is of full rank, (A14) and (A21) imply

$$\text{rank}[M(1)] = \text{rank}[M^R(1)M(0)] = \text{rank}[M^R(1)] = n - r.$$

This gives (A18).

Premultiplying (A15) by $M^{-1}(0)$ and using $\Pi(0) = A(0) = M^{-1}(0)$, where $A(0)$ is the leading parameter matrix of the structural vector autoregressive representation, one obtains (A19), where

$$\lambda = M^{-1}(0)\lambda^R. \quad (\text{A23})$$

Finally, because of (A16), (A21) and (A23), one obtains

$$M(1)\lambda = M^R(1)M(0)M^{-1}(0)\lambda^R = M^R(1)\lambda^R = 0_{n \times r}.$$

This gives (A20). ■

Lau (1999) points out a systematic difference between endogenous and exogenous growth models in terms of the structural long-run multiplier matrix, when the variables are cointegrated. Using Lemma A-1, it can be shown that the difference also appears in the structural adjustment matrix. This is given in the following Proposition.

Proposition A-2. *If the last r columns of the structural long-run multiplier matrix are zero (and the other columns are non-zero) for the cointegrated system (A17), then the first $n - r$ rows of the structural adjustment matrix*

are zero. That is, if λ in (A19) is partitioned as $\lambda = (\lambda'_a, \lambda'_b)'$ such that λ_a is $(n-r) \times r$ and λ_b is $r \times r$, then λ_b is non-zero and

$$\lambda_a = 0_{(n-r) \times r}. \quad (\text{A24})$$

Proof. Order the n variables such that the last r columns of the structural long-run multiplier matrix are zero. That is,

$$M(1) = (D, 0_{n \times r}), \quad (\text{A25})$$

where D is a $n \times (n-r)$ matrix with rank $n-r$. Therefore, (A20) and (A25) imply

$$0_{n \times r} = M(1) \lambda = (D, 0) \begin{pmatrix} \lambda_a \\ \lambda_b \end{pmatrix} = D \lambda_a. \quad (\text{A26})$$

Since D is of rank $n-r$, there are only $n-r$ independent rows. Select any $n-r$ independent rows from D and call it D_1 . It is easy to see that D_1 is a square matrix with full rank; therefore, its inverse exists. Out of the $n \times r$ equations in (A26), the $(n-r) \times r$ equations corresponding to the $n-r$ rows forming D_1 are given by

$$0_{(n-r) \times r} = D_1 \lambda_a. \quad (\text{A27})$$

Since D_1 is of full rank, premultiplying (A27) by the inverse of D_1 leads to (A24). ■

Consider the reduced-form vector error-correction model (A15). Partition λ^R as $\lambda^R = \left((\lambda_a^R)', (\lambda_b^R)' \right)'$ such that λ_a^R is $(n-r) \times r$ and λ_b^R is $r \times r$. Partition M_0 , the leading structural VMA parameter matrix, as

$$M_0 \equiv M(0) = \begin{bmatrix} M_{0,aa} & M_{0,ab} \\ M_{0,ba} & M_{0,bb} \end{bmatrix},$$

where $M_{0,aa}$ is $(n-r) \times (n-r)$, $M_{0,ab}$ is $(n-r) \times r$, $M_{0,ba}$ is $r \times (n-r)$, and $M_{0,bb}$ is $r \times r$. The following Proposition shows that, under certain identifying assumptions, Proposition A-2 above leads to testable implications on the reduced-form vector error-correction model.

Proposition A-3. *If the identifying assumptions*

$$M_{0,ab} = 0_{(n-r) \times r} \quad (\text{A28})$$

are applied to the reduced-form vector error-correction model (A15), then testing whether

$$\lambda_a^R = 0_{(n-r) \times r} \quad (\text{A29})$$

can be used to test whether the long-run effect of the last r structural disturbances is zero or not.

Proof. Using the relationship between the reduced-form and structural models, we know from (A23) that

$$\lambda^R = M_0 \lambda.$$

Therefore,

$$\begin{pmatrix} \lambda_a^R \\ \lambda_b^R \end{pmatrix} = \begin{pmatrix} M_{0,aa} \lambda_a + M_{0,ab} \lambda_b \\ M_{0,ba} \lambda_a + M_{0,bb} \lambda_b \end{pmatrix}. \quad (\text{A30})$$

Under the identifying assumptions (A28), one can use (A30) to conclude that (A24) implies (A29). ■

Proposition A-3 is particularly useful when $r = 1$ (with $n \geq 2$), as the zero restrictions in (A28) are the same as a subset of the restrictions imposed by Sims' recursive ordering. When $n = 2$, the identifying assumption (A28) corresponds exactly to Sims' x_1 -first ordering (i.e., $m_{0,12} = 0$) and the system is just identified. In this case, Proposition A-3 becomes Proposition 1 in the main text. When $n \geq 3$, the identifying assumptions (A28) are not enough to identify the system.²¹ However, Proposition A-3 may still be useful if the system is identified by combining (A28) with other identifying assumptions. The application of Proposition A-3 to higher-dimensional systems is left to future studies.

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²¹ For a system of n variables, one needs at least $0.5n(n-1)$ restrictions for identification; see, for example, Hamilton (1994, p. 332). When $r = 1$, the identifying assumptions (A28) impose $(n-r) \times r = n-1$ restrictions. It is straightforward to show that $0.5n(n-1) - (n-1) = 0.5(n-1)(n-2)$, which is always positive for $n \geq 3$.

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Table 1: AIC, BIC and Durbin-Watson Statistics

Country	Lag length	AIC	BIC	DW(Δly)	DW(Δli)
Canada	1	-7.99	-7.67	1.73	1.82
Canada	2	-7.89	-7.41	1.76	1.81
Canada	3	-7.83	-7.19	1.93	2.05
Canada	4	-7.75	-6.96	1.88	1.99
France	1	-9.05	-8.73	1.95	1.97
France	2	-8.92	-8.45	1.94	1.95
France	3	-8.86	-8.22	2.15	2.00
France	4	-8.89	-8.10	2.03	2.01
Italy	1	-8.86	-8.54	2.04	1.99
Italy	2	-8.88	-8.40	1.93	1.95
Italy	3	-8.71	-8.08	1.96	1.95
Italy	4	-8.72	-7.93	1.99	1.98
Japan	1	-8.44	-8.13	2.37	2.03
Japan	2	-8.54	-8.06	2.11	2.09
Japan	3	-8.47	-7.83	2.12	2.08
Japan	4	-8.38	-7.58	2.16	2.08
UK	1	-8.49	-8.17	1.76	1.69
UK	2	-8.56	-8.09	2.03	1.94
UK	3	-8.42	-7.78	1.98	1.92
UK	4	-8.42	-7.63	2.01	1.98
USA	1	-8.23	-7.91	1.76	1.87
USA	2	-8.26	-7.79	1.82	1.91
USA	3	-8.18	-7.55	1.79	1.87
USA	4	-8.12	-7.32	1.77	1.81

Note: AIC represents Akaike Information Criterion, BIC represents Bayesian Information Criterion, DW represents the Durbin-Watson statistic, and ly and li represent log per capita GDP and log per capita investment respectively.

Table 2: Likelihood Ratio Tests for the Null Hypothesis of No Cointegration

Country	Lag length	L_U	L_R	$LR = 2(L_U - L_R)$
Canada	1	191.8	189.0	5.6
France	1	216.1	208.3	15.6***
Italy	2	216.2	208.2	16.0***
Japan	2	208.3	199.5	17.6***
UK	2	208.9	204.4	9.0*
USA	2	202.0	201.8	0.4

Note: The number of lags is determined by the Akaike Information Criterion. LR stands for the likelihood ratio test statistic for the hypothesis $\lambda_1^R = \lambda_2^R = 0$ in (8). The critical value of the LR test statistic for the null hypothesis of no cointegration is 8.30 at 10% significance level, 10.18 at 5% significance level, and 13.73 at 1% significance level respectively (Horvath and Watson, 1995, Table 1). The symbols *, ** and *** represent statistical significance at 10%, 5%, and 1% respectively.

Table 3: Estimated Coefficients and t -statistics for France, Italy, Japan and UK

Country		France		Italy		Japan		UK	
Left-hand variable		Δly_t	Δli_t	Δly_t	Δli_t	Δly_t	Δli_t	Δly_t	Δli_t
Right-hand variables	constant	-0.09 (-2.43)	-0.54 (-3.98)	0.03 (0.57)	-0.22 (-1.09)	-0.07 (-3.03)	-0.25 (-4.28)	-0.02 (-0.44)	-0.38 (-2.08)
	$(li - ly)_{t-1}$	-0.068 (-2.79)	-0.36 (-4.02)	0.017 (0.49)	-0.14 (-1.12)	-0.056 (-3.42)	-0.19 (-4.31)	-0.023 (-0.95)	-0.24 (-2.34)
	Δly_{t-1}	0.93 (4.01)	2.82 (3.33)	0.61 (1.73)	1.53 (1.23)	0.87 (3.54)	2.21 (3.38)	0.49 (1.83)	2.19 (1.93)
	Δly_{t-2}			0.31 (0.85)	0.64 (0.50)	0.44 (1.90)	0.46 (0.76)	-0.46 (-1.73)	-2.07 (-1.84)
	Δli_{t-1}	-0.15 (-2.44)	-0.53 (-2.36)	-0.14 (-1.50)	-0.27 (-0.83)	-0.16 (-1.75)	-0.42 (-1.76)	-0.02 (-0.38)	-0.06 (-0.26)
	Δli_{t-2}			-0.12 (-1.21)	-0.14 (-0.40)	-0.14 (-1.75)	-0.35 (-1.61)	0.03 (0.53)	0.03 (0.12)
$M_{11}(1)$		0.015		0.116		0.027		0.012	
$M_{12}(1)$		-0.014		0.014		-0.035		-0.004	

Note: The sample period is 1955 to 2000. Numbers without parentheses are the estimated coefficients and numbers inside parentheses represent t -statistics.

Table 4: Cointegration Tests for Various Models

(A) Based on per capita output and per capita investment

Country	Lag length	L_U	L_R	$LR = 2(L_U - L_R)$
Italy	1	211.7	204.7	14.0 ^{***}
Japan	1	202.2	194.4	15.6 ^{***}
UK	1	203.2	197.0	12.4 ^{**}
USA	1	197.3	196.7	1.2

(B) Based on output per worker and investment per worker

Country	Lag length	L_U	L_R	$LR = 2(L_U - L_R)$
Canada	1	188.2	186.2	4.0
France	1	216.2	209.9	12.6 ^{**}
Italy	2	213.8	205.2	17.2 ^{***}
Japan	2	209.4	200.4	18.0 ^{***}
UK	2	209.6	205.0	9.2 [*]
USA	2	198.2	197.7	1.0

Note: See Table 2.

Table 5: Estimated Coefficients and Long-run Responses for Various Models

(A) Based on per capita output and per capita investment

Country	Lag length	λ_1^R	λ_2^R	$M_{12}(1)$
France	1	-0.068 (-2.79)	-0.36 (-4.02)	-0.014
France	2	-0.070 (-2.64)	-0.36 (-3.70)	-0.014
France	3	-0.072 (-2.76)	-0.36 (-3.76)	-0.018
France	4	-0.072 (-2.71)	-0.36 (-3.60)	-0.028
Italy	1	0.017 (0.58)	-0.11 (-1.06)	0.013
Italy	2	0.017 (0.49)	-0.14 (-1.12)	0.014
Italy	3	0.022 (0.51)	-0.14 (-0.91)	0.016
Italy	4	0.035 (0.74)	-0.15 (-0.90)	0.012
Japan	1	-0.052 (-3.17)	-0.18 (-4.11)	-0.029
Japan	2	-0.056 (-3.42)	-0.19 (-4.31)	-0.035
Japan	3	-0.060 (-3.51)	-0.20 (-4.41)	-0.029
Japan	4	-0.066 (-3.76)	-0.21 (-4.59)	-0.029
UK	1	-0.037 (-1.54)	-0.32 (-3.03)	-0.007
UK	2	-0.023 (-0.95)	-0.24 (-2.34)	-0.004
UK	3	-0.019 (-0.75)	-0.23 (-2.19)	-0.003
UK	4	-0.012 (-0.45)	-0.23 (-2.11)	-0.001

(B) Based on output per worker and investment per worker

Country	Lag length	λ_1^R	λ_2^R	$M_{12}(1)$
France	1	-0.076 (-3.17)	-0.32 (-3.62)	-0.022
Italy	2	0.003 (0.07)	-0.20 (-1.46)	0.003
Japan	2	-0.060 (-3.52)	-0.20 (-4.35)	-0.036
UK	2	-0.031 (-1.21)	-0.25 (-2.40)	-0.005

Note: Numbers before parentheses are the estimated coefficients and numbers inside parentheses are t -statistics. The long-run responses $M_{12}(1)$ are obtained under the identifying assumption of $m_{0,12} = 0$.

Figure 1: Canada

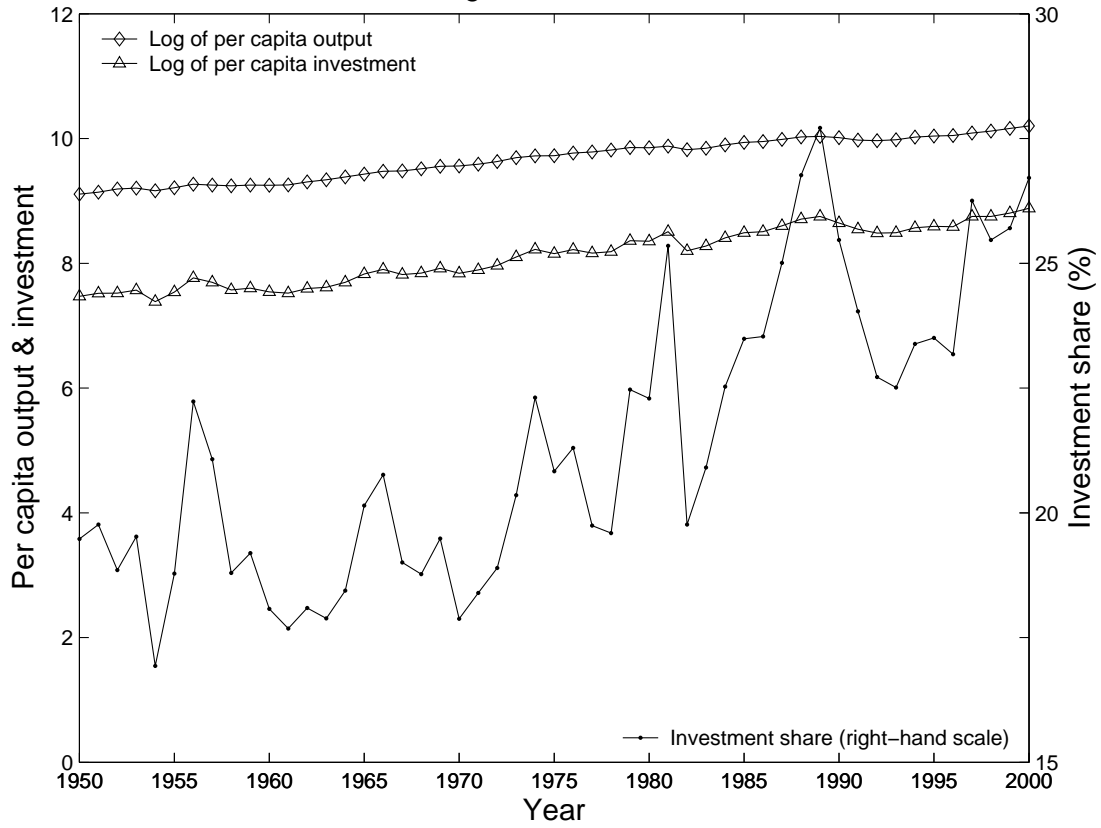


Figure 2: France

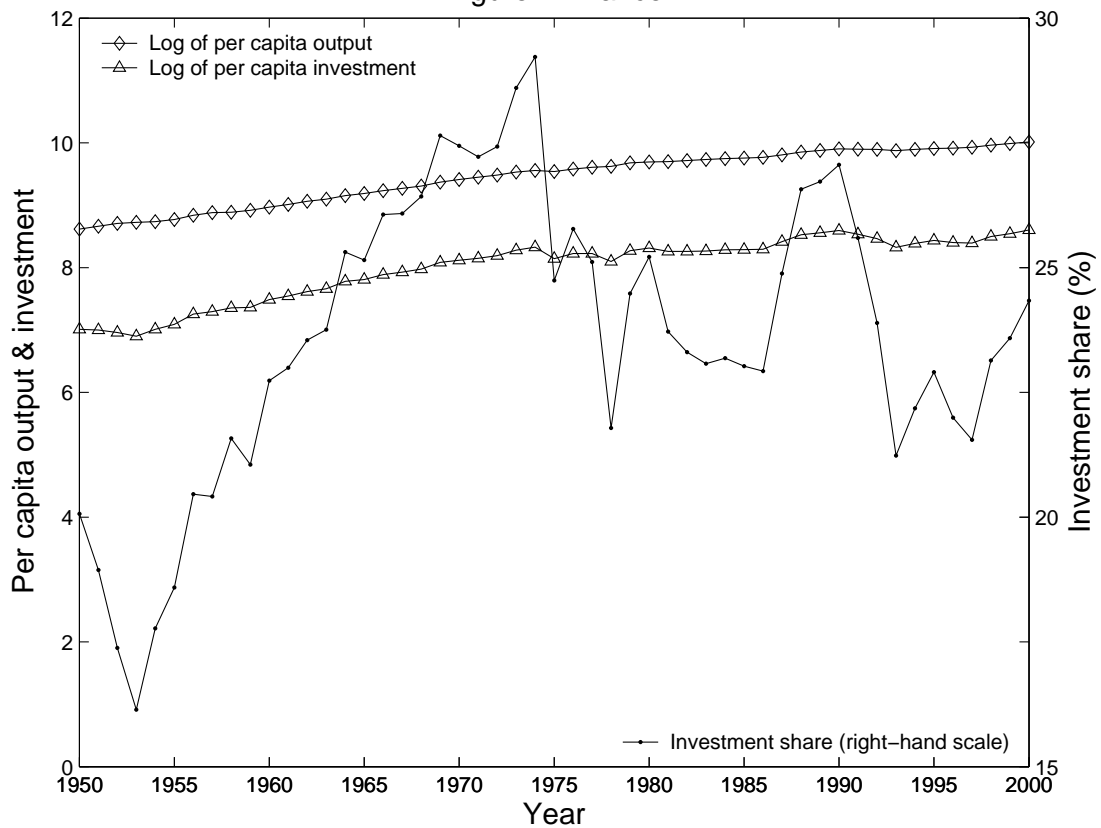


Figure 3: Italy

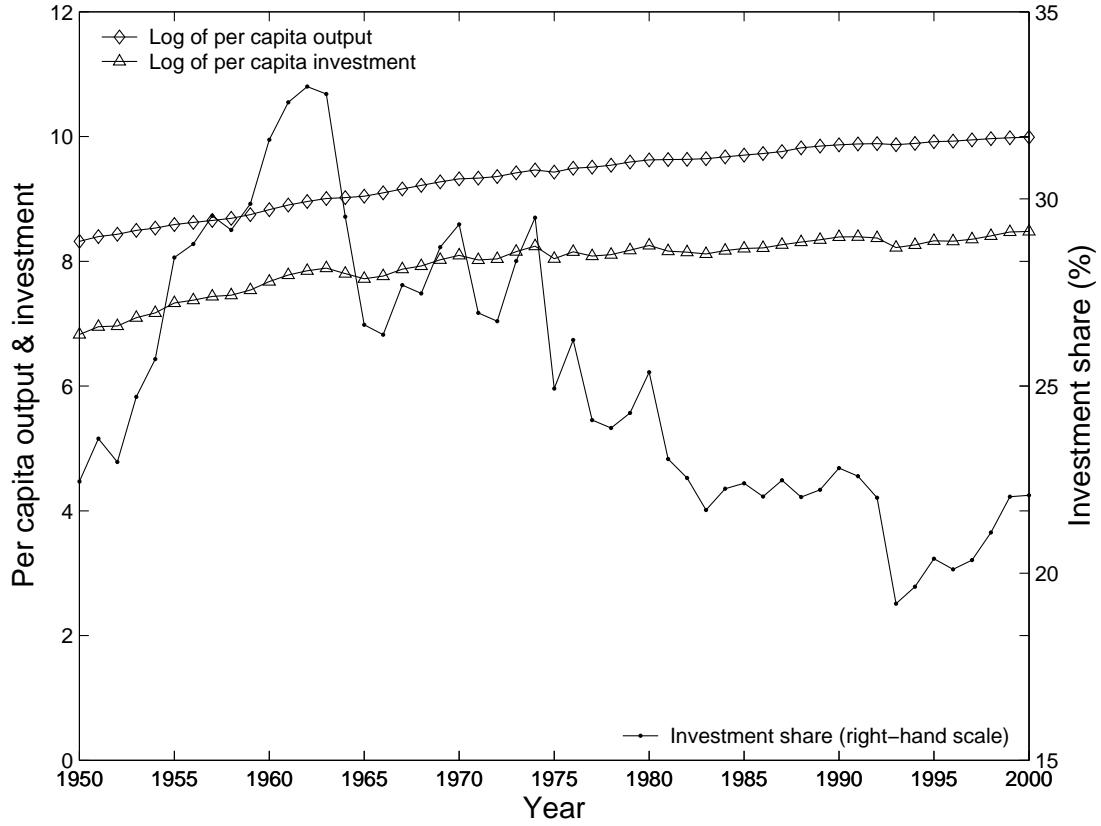


Figure 4: Japan

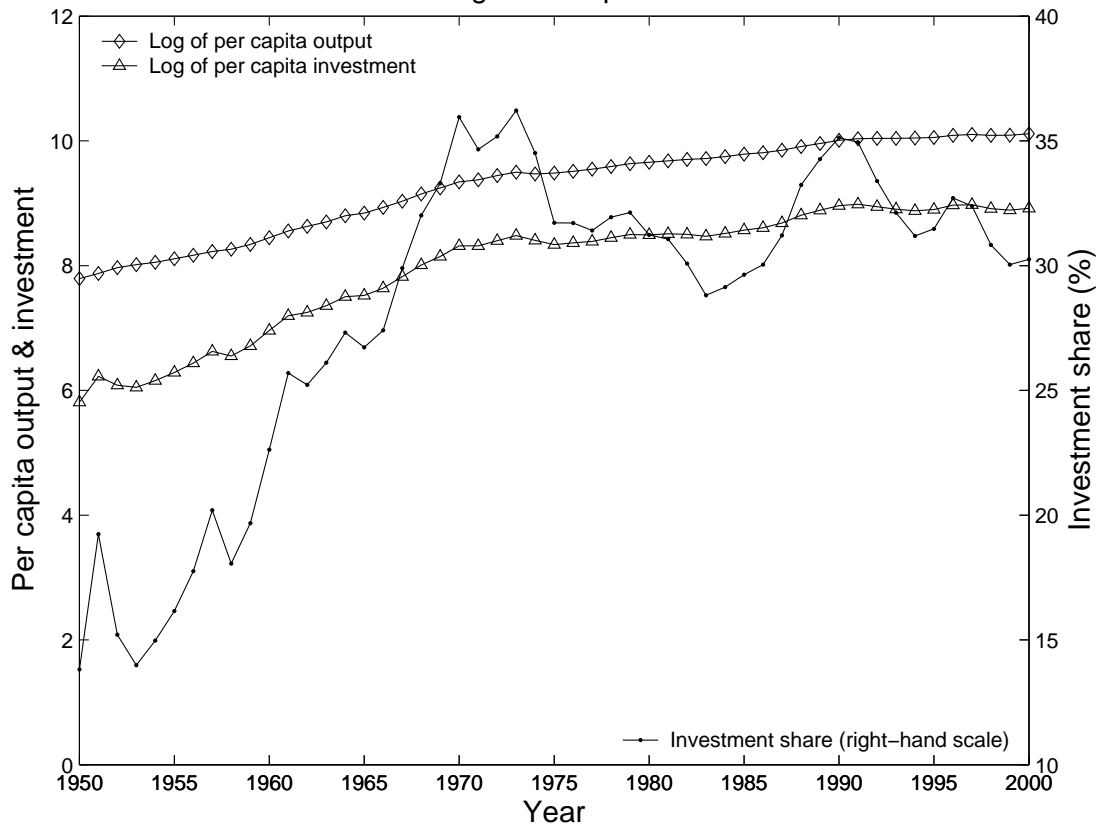


Figure 5: UK

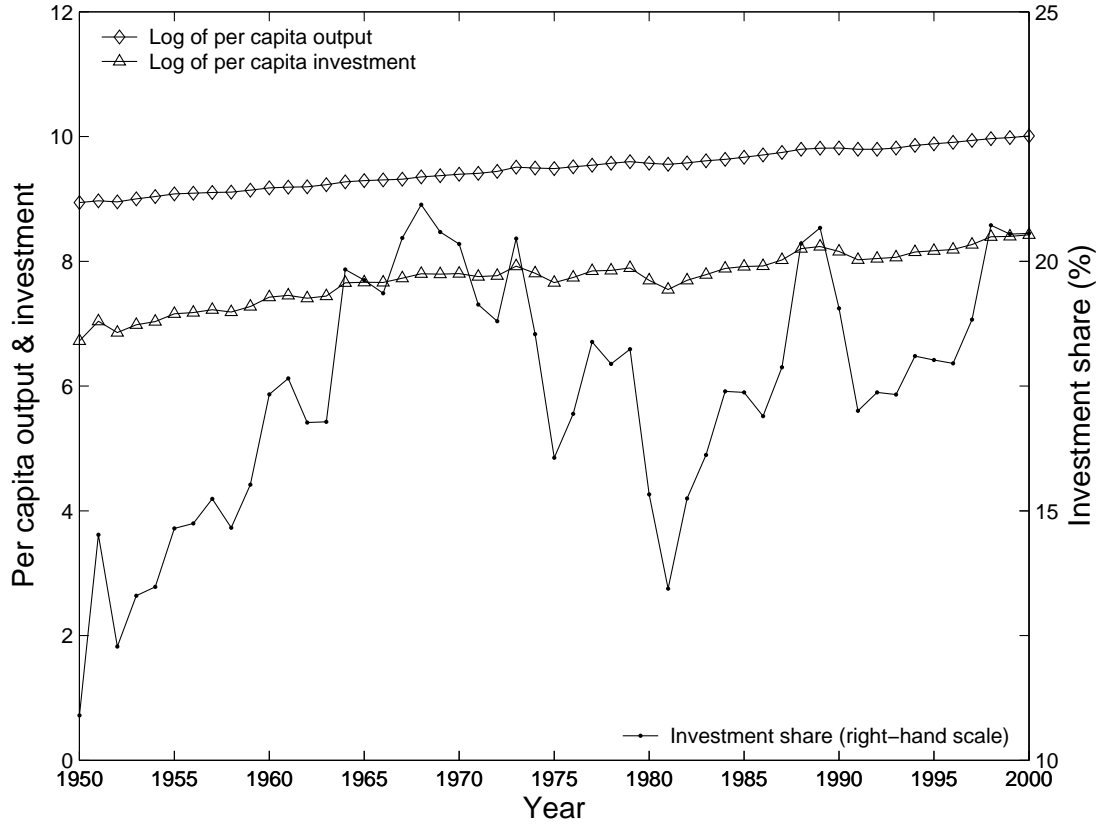


Figure 6: USA

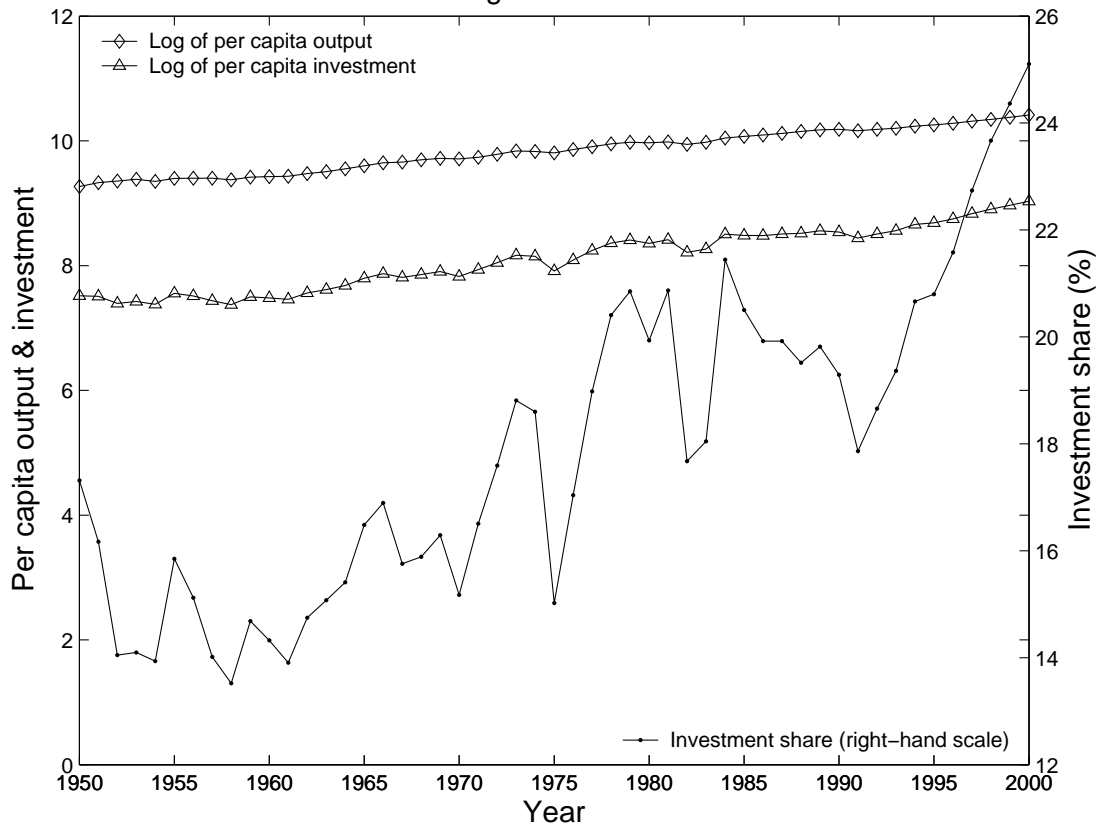


Figure 7: CUSUM Analysis for France

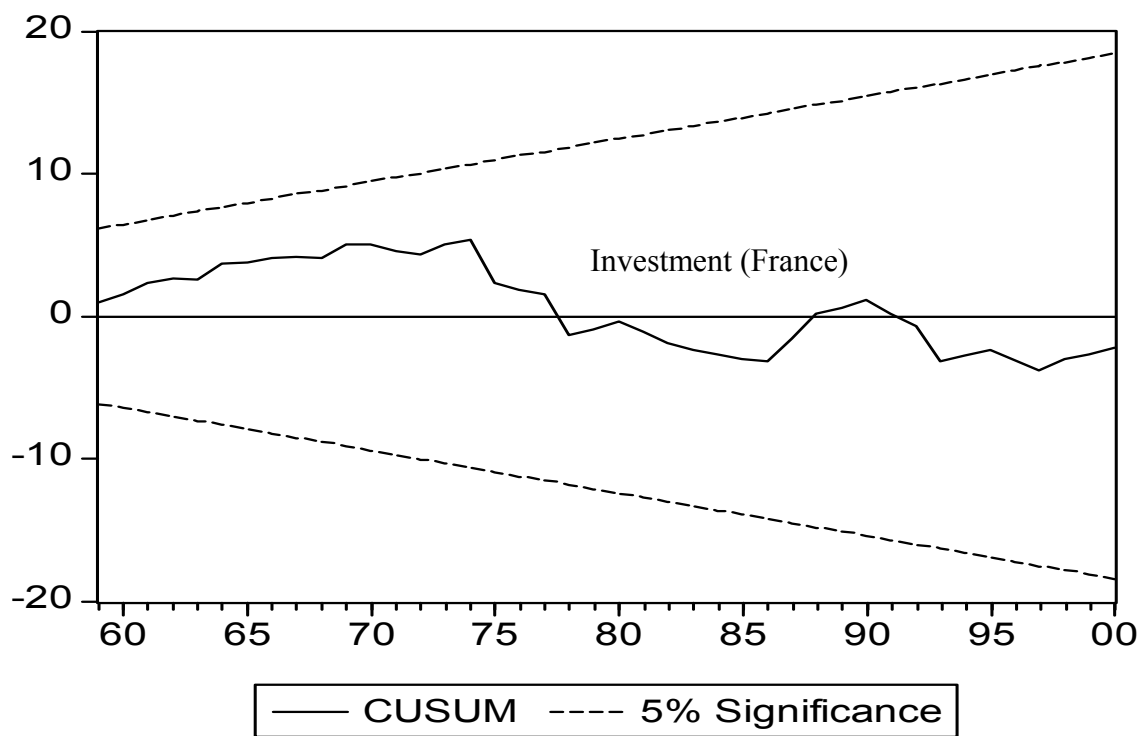
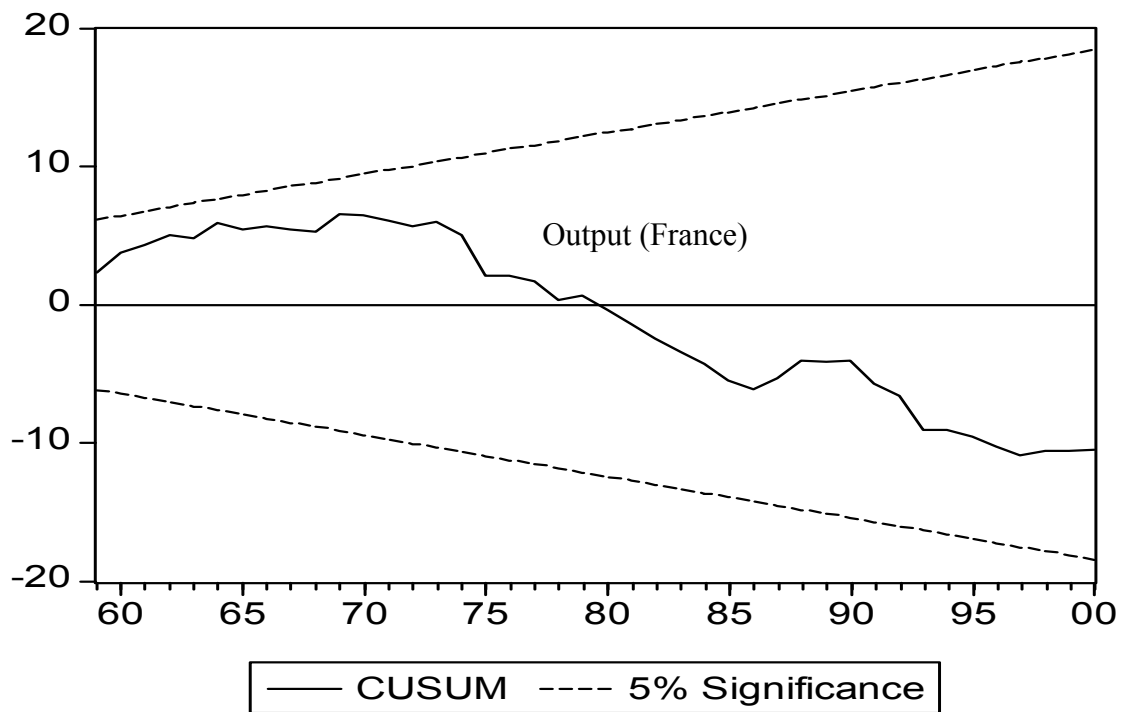


Figure 8: CUSUM Analysis for Italy

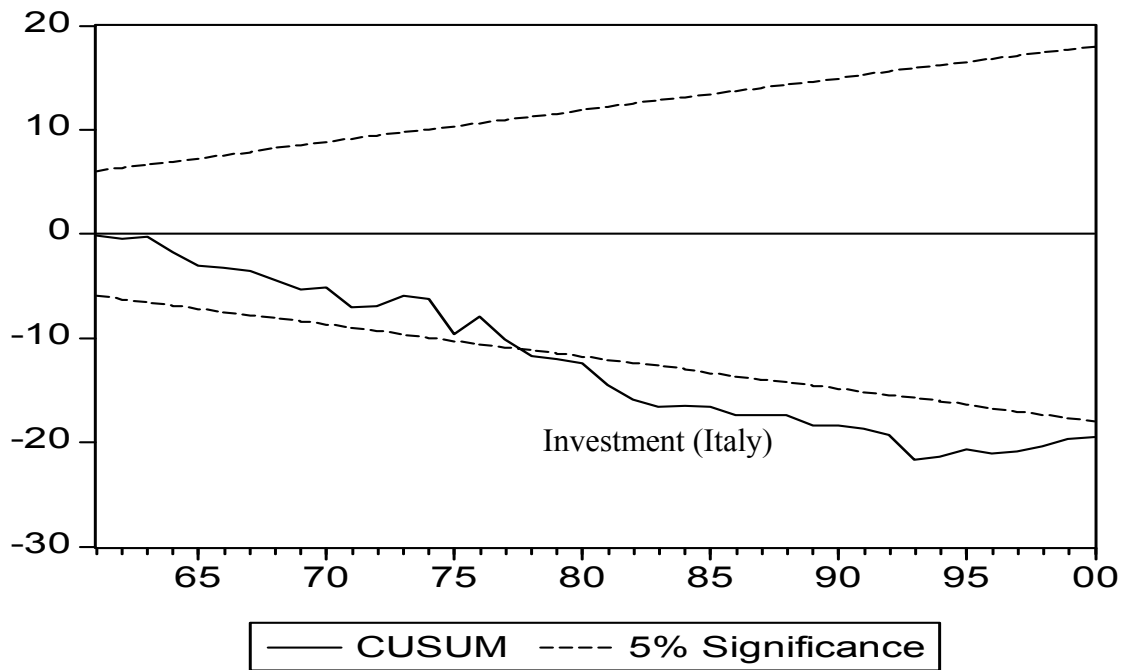
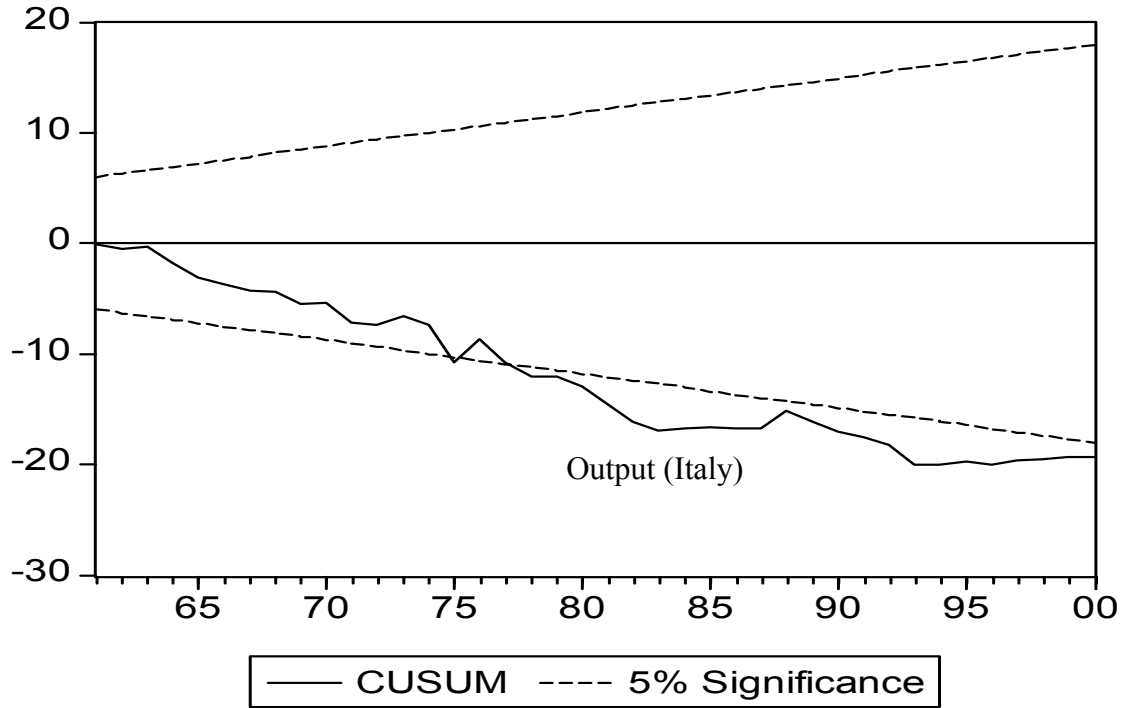


Figure 9: CUSUM Analysis for Japan

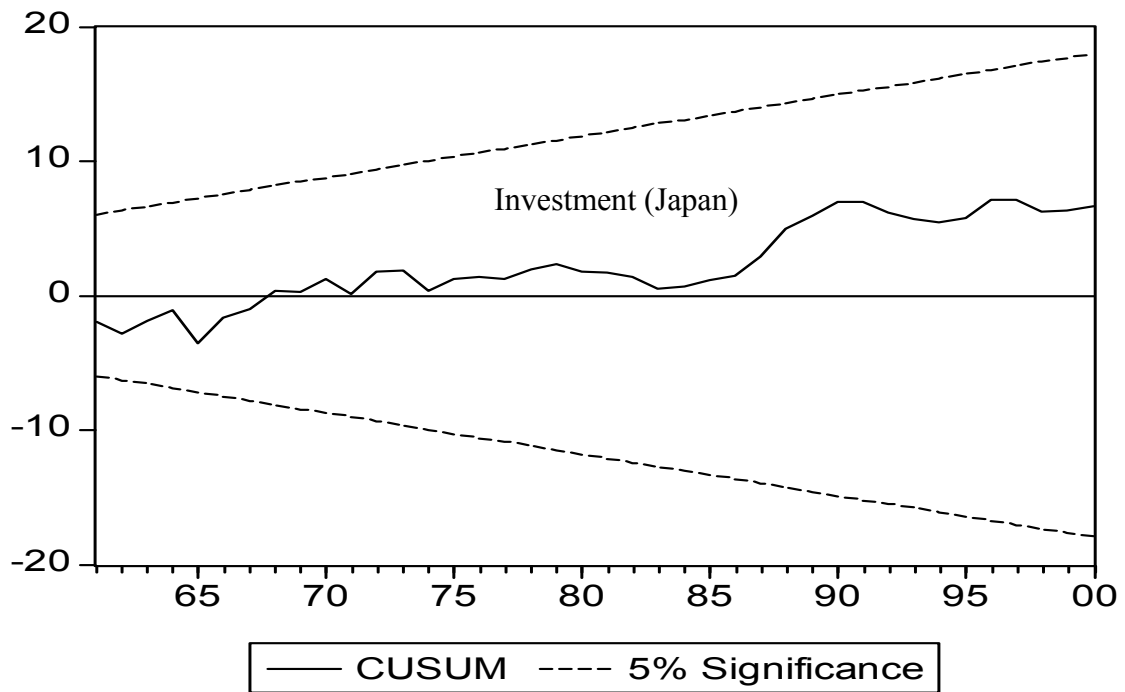
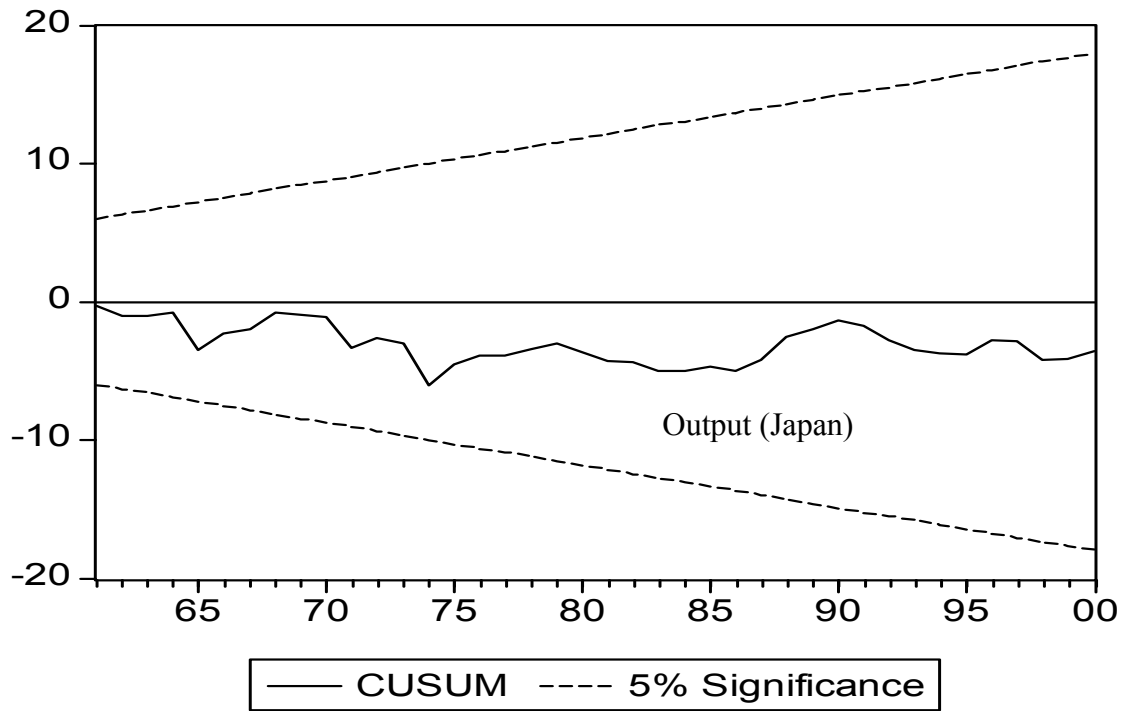


Figure 10: CUSUM Analysis for UK

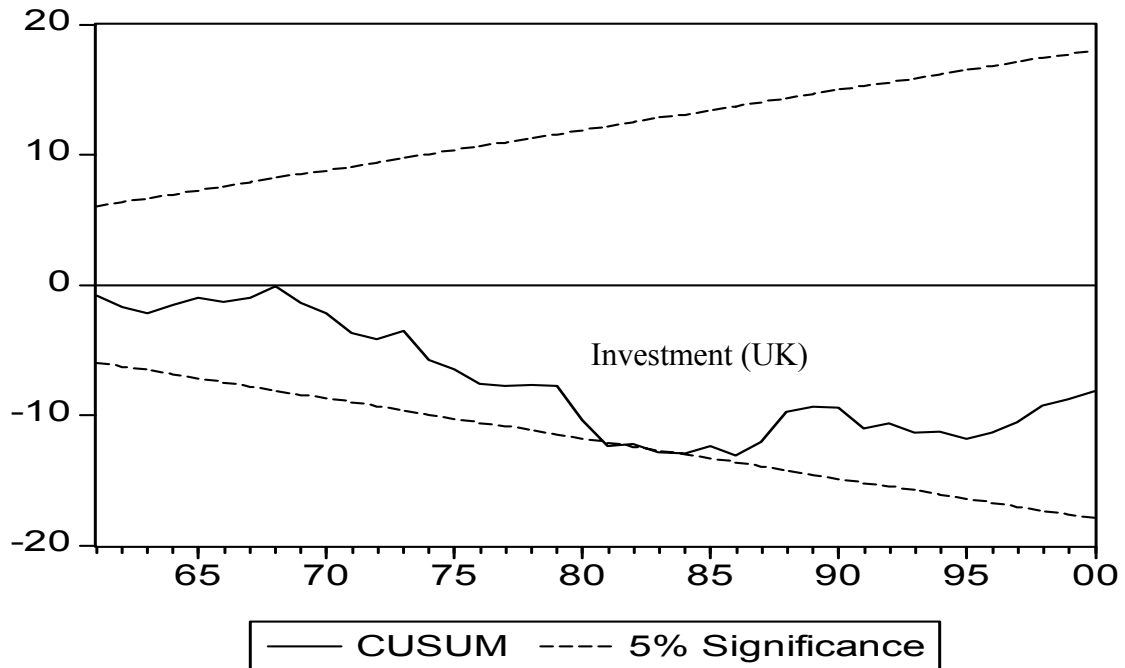
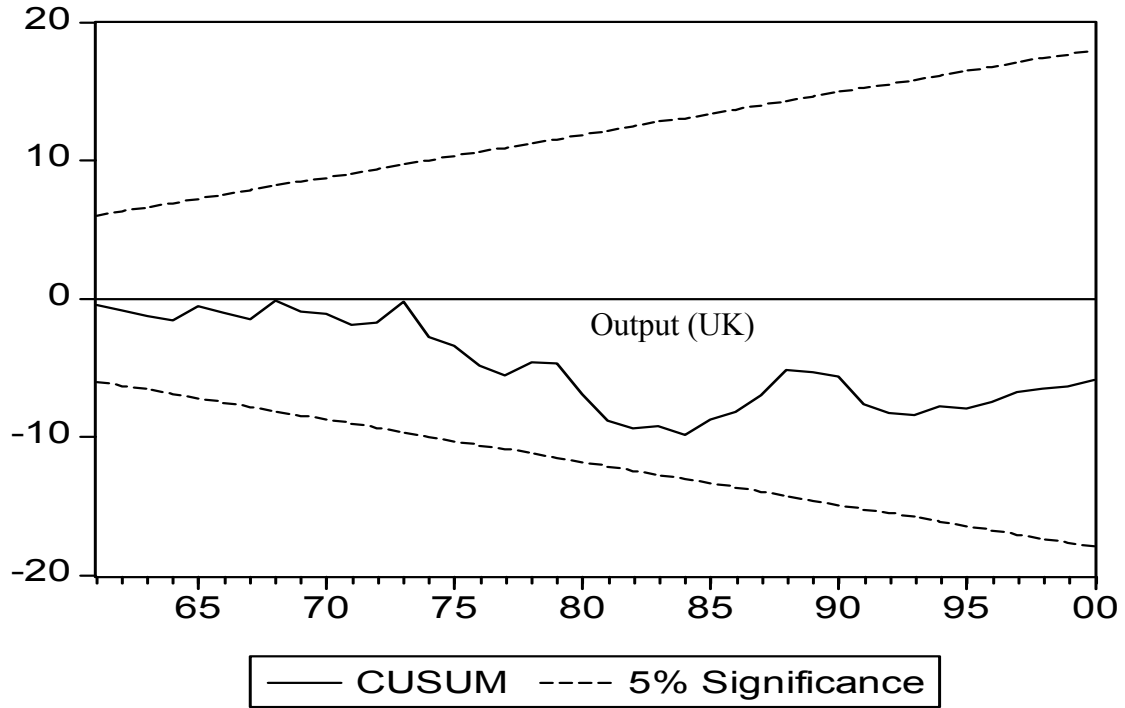


Figure A1: Analysis of Residuals (Canada)

Correlogram of Residuals

Canada: Output Equation Sample: 1955 2000						
Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob	
		1	0.112	0.112	0.6204	0.431
		2	-0.129	-0.143	1.4517	0.484
		3	-0.010	0.023	1.4572	0.692
		4	0.076	0.058	1.7585	0.780
		5	-0.120	-0.140	2.5277	0.772
		6	-0.181	-0.136	4.3356	0.631
		7	-0.062	-0.059	4.5534	0.714
		8	0.036	0.004	4.6300	0.796
		9	0.062	0.062	4.8622	0.846
		10	0.128	0.134	5.8600	0.827
		11	0.141	0.108	7.1053	0.790
		12	0.052	0.022	7.2797	0.839
		13	0.122	0.139	8.2791	0.825
		14	-0.005	-0.024	8.2811	0.874
		15	-0.075	-0.012	8.6822	0.894
		16	-0.055	0.019	8.9040	0.917
		17	-0.080	-0.064	9.3951	0.927
		18	-0.066	-0.022	9.7423	0.940
		19	0.034	0.054	9.8385	0.957
		20	-0.061	-0.127	10.151	0.965

Correlogram of Residuals

Canada: Investment equation Sample: 1955 2000						
Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob	
		1	0.072	0.072	0.2576	0.612
		2	-0.121	-0.127	0.9898	0.610
		3	-0.010	0.009	0.9949	0.802
		4	-0.020	-0.036	1.0167	0.907
		5	0.023	0.028	1.0459	0.959
		6	-0.115	-0.129	1.7754	0.939
		7	0.061	0.092	1.9853	0.961
		8	0.109	0.065	2.6724	0.953
		9	0.128	0.142	3.6479	0.933
		10	0.143	0.141	4.8983	0.898
		11	-0.095	-0.075	5.4733	0.906
		12	-0.046	-0.012	5.6095	0.934
		13	0.078	0.084	6.0177	0.946
		14	-0.038	-0.045	6.1191	0.963
		15	-0.042	-0.016	6.2445	0.975
		16	0.001	0.003	6.2446	0.985
		17	-0.082	-0.164	6.7545	0.987
		18	0.000	-0.027	6.7545	0.992
		19	0.052	0.032	6.9740	0.994
		20	-0.115	-0.162	8.0972	0.991

Figure A2: Analysis of Residuals (France)

Correlogram of Residuals

France: Output Equation Sample: 1955 2000						
Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob	
		1	0.018	0.018	0.0155	0.901
		2	-0.026	-0.026	0.0485	0.976
		3	0.106	0.107	0.6264	0.890
		4	-0.002	-0.007	0.6267	0.960
		5	0.120	0.127	1.3964	0.925
		6	-0.162	-0.185	2.8508	0.827
		7	0.049	0.075	2.9866	0.886
		8	0.026	-0.024	3.0252	0.933
		9	-0.041	0.009	3.1261	0.959
		10	0.105	0.077	3.8047	0.956
		11	-0.111	-0.084	4.5813	0.950
		12	0.006	-0.014	4.5837	0.970
		13	0.043	0.032	4.7065	0.981
		14	-0.004	0.022	4.7077	0.989
		15	-0.046	-0.080	4.8567	0.993
		16	-0.089	-0.034	5.4340	0.993
		17	0.156	0.124	7.2971	0.979
		18	0.036	0.031	7.3997	0.986
		19	-0.090	-0.058	8.0597	0.986
		20	-0.007	-0.034	8.0645	0.991

Correlogram of Residuals

France: Investment Equation Sample: 1955 2000						
Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob	
		1	0.010	0.010	0.0050	0.944
		2	-0.133	-0.133	0.8907	0.641
		3	0.146	0.152	1.9920	0.574
		4	-0.055	-0.083	2.1524	0.708
		5	-0.129	-0.088	3.0530	0.692
		6	-0.203	-0.250	5.3334	0.502
		7	-0.011	-0.011	5.3403	0.619
		8	0.051	0.018	5.4935	0.704
		9	-0.250	-0.226	9.2181	0.417
		10	-0.022	-0.058	9.2474	0.509
		11	-0.054	-0.218	9.4283	0.582
		12	-0.106	-0.122	10.158	0.602
		13	0.027	-0.077	10.205	0.677
		14	0.006	-0.078	10.208	0.747
		15	0.164	0.045	12.133	0.669
		16	-0.029	-0.177	12.194	0.731
		17	-0.035	-0.103	12.288	0.782
		18	0.231	0.046	16.491	0.558
		19	-0.031	-0.064	16.567	0.619
		20	-0.024	-0.030	16.617	0.678

Figure A3: Analysis of Residuals (Italy)

Correlogram of Residuals

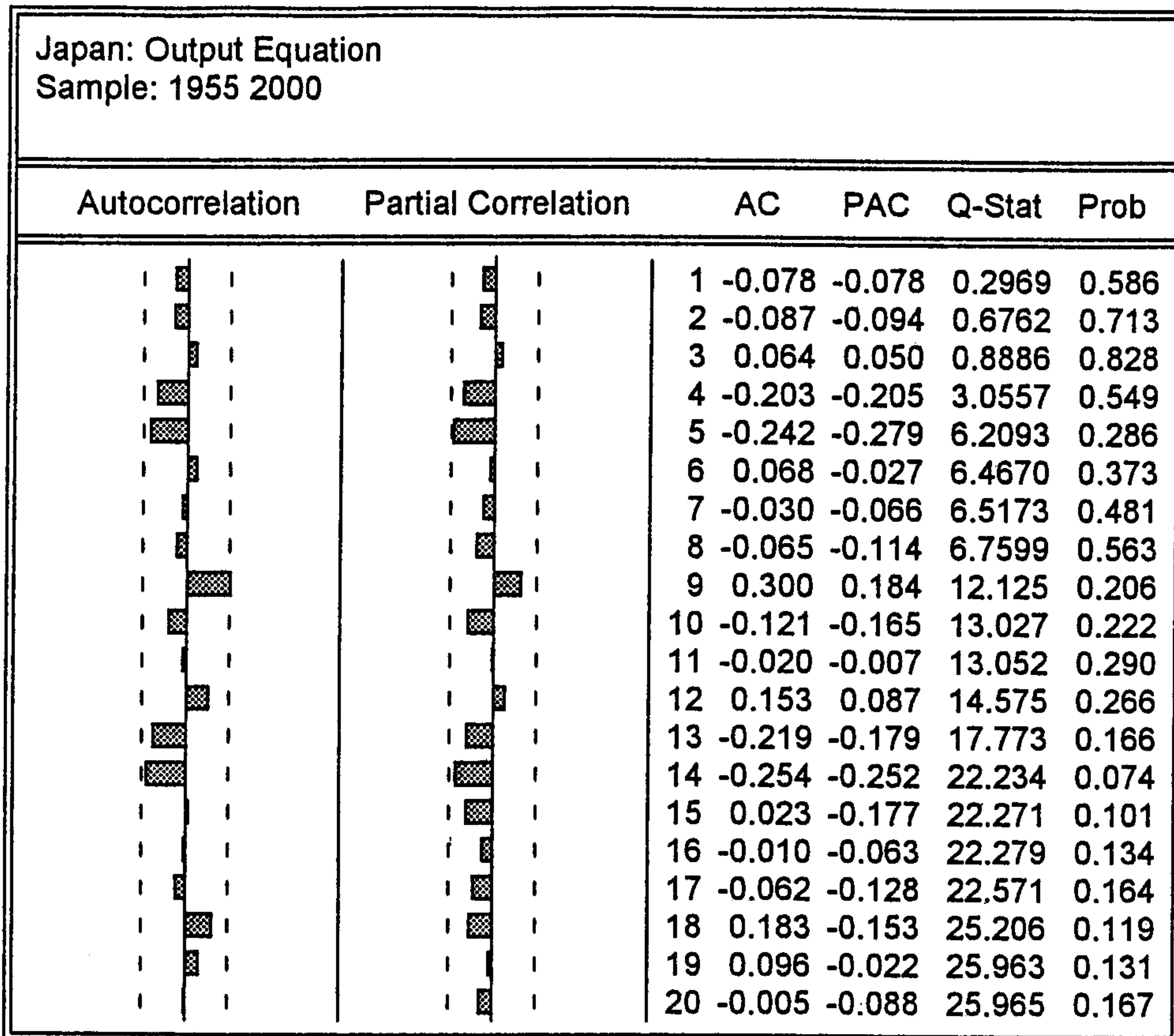
Italy: Output Equation Sample: 1955 2000						
Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob	
		1	0.003	0.003	0.0004	0.983
		2	0.009	0.009	0.0045	0.998
		3	0.074	0.074	0.2864	0.963
		4	-0.069	-0.070	0.5365	0.970
		5	-0.200	-0.202	2.6814	0.749
		6	0.109	0.110	3.3320	0.766
		7	0.027	0.046	3.3723	0.849
		8	0.003	0.022	3.3726	0.909
		9	-0.055	-0.108	3.5537	0.938
		10	0.057	0.030	3.7532	0.958
		11	0.077	0.140	4.1262	0.966
		12	0.111	0.135	4.9268	0.960
		13	-0.069	-0.117	5.2470	0.969
		14	-0.055	-0.132	5.4557	0.978
		15	-0.004	0.049	5.4566	0.988
		16	-0.066	0.037	5.7760	0.990
		17	-0.064	-0.053	6.0881	0.993
		18	0.205	0.109	9.4158	0.949
		19	0.067	0.072	9.7809	0.958
		20	-0.212	-0.203	13.597	0.850

Correlogram of Residuals

Italy: Investment Equation Sample: 1955 2000						
Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob	
		1	-0.015	-0.015	0.0114	0.915
		2	-0.032	-0.032	0.0613	0.970
		3	0.074	0.073	0.3443	0.951
		4	0.102	0.104	0.8941	0.925
		5	-0.194	-0.189	2.9238	0.712
		6	0.164	0.169	4.4028	0.622
		7	0.079	0.055	4.7523	0.690
		8	-0.141	-0.130	5.9116	0.657
		9	-0.204	-0.196	8.4031	0.494
		10	0.060	-0.016	8.6260	0.568
		11	0.142	0.234	9.8894	0.540
		12	0.098	0.175	10.511	0.571
		13	0.025	-0.024	10.554	0.648
		14	0.008	-0.090	10.558	0.720
		15	-0.016	0.026	10.576	0.782
		16	-0.018	0.051	10.600	0.834
		17	-0.026	-0.115	10.651	0.874
		18	0.182	0.086	13.250	0.776
		19	-0.014	0.037	13.266	0.825
		20	-0.302	-0.225	21.023	0.396

Figure A4: Analysis of Residuals (Japan)

Correlogram of Residuals



Correlogram of Residuals

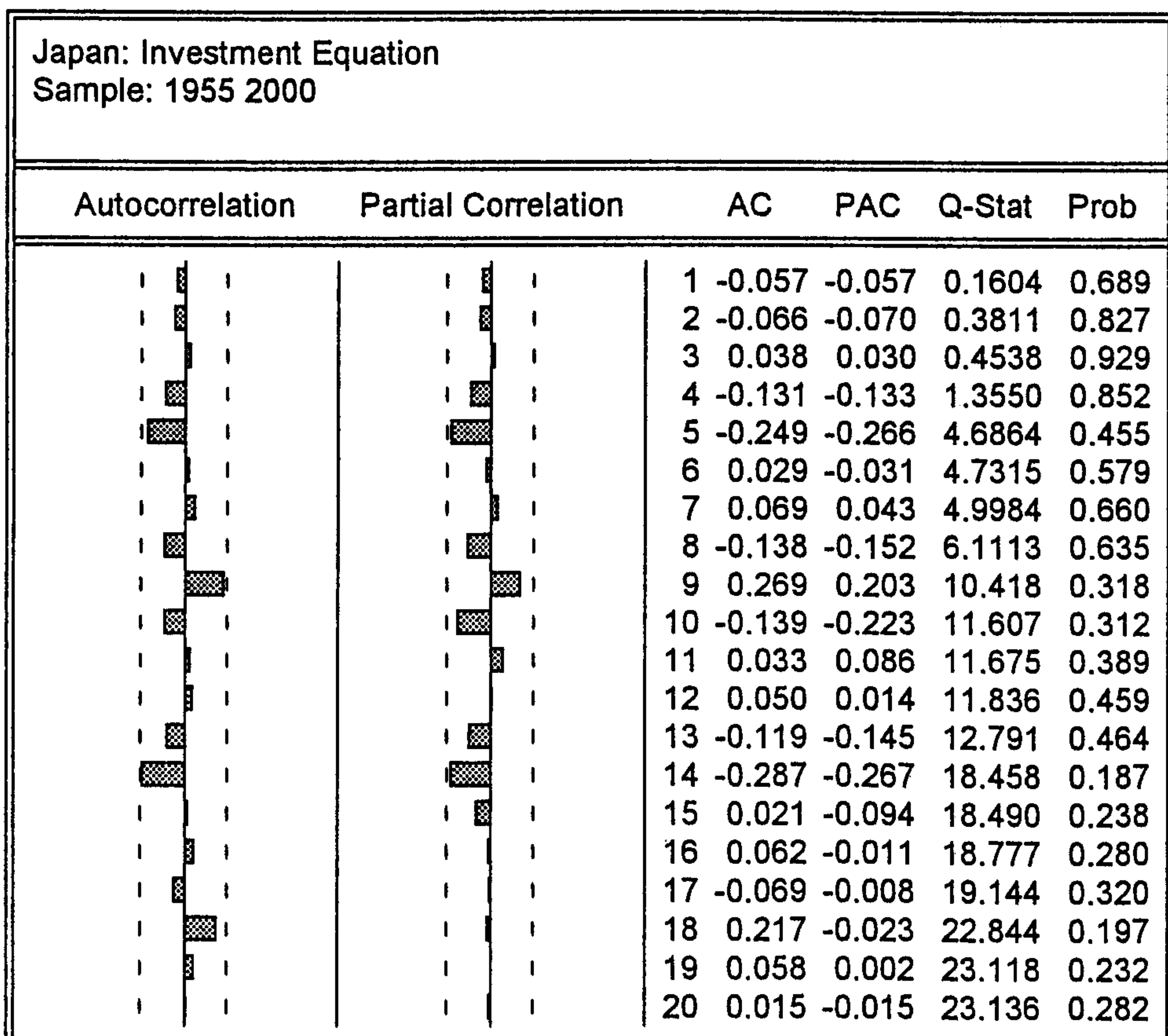
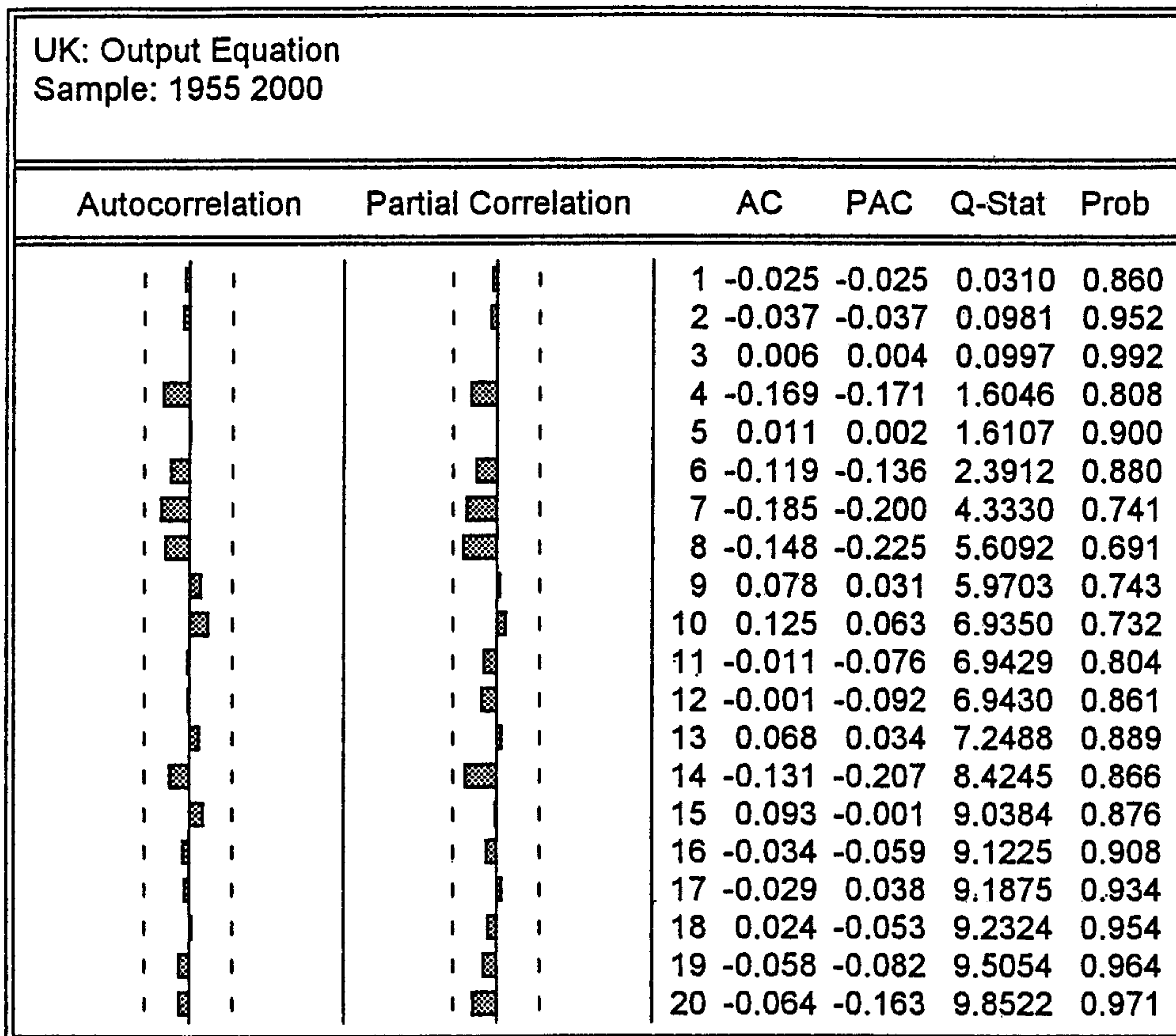


Figure A5: Analysis of Residuals (UK)

Correlogram of Residuals



Correlogram of Residuals

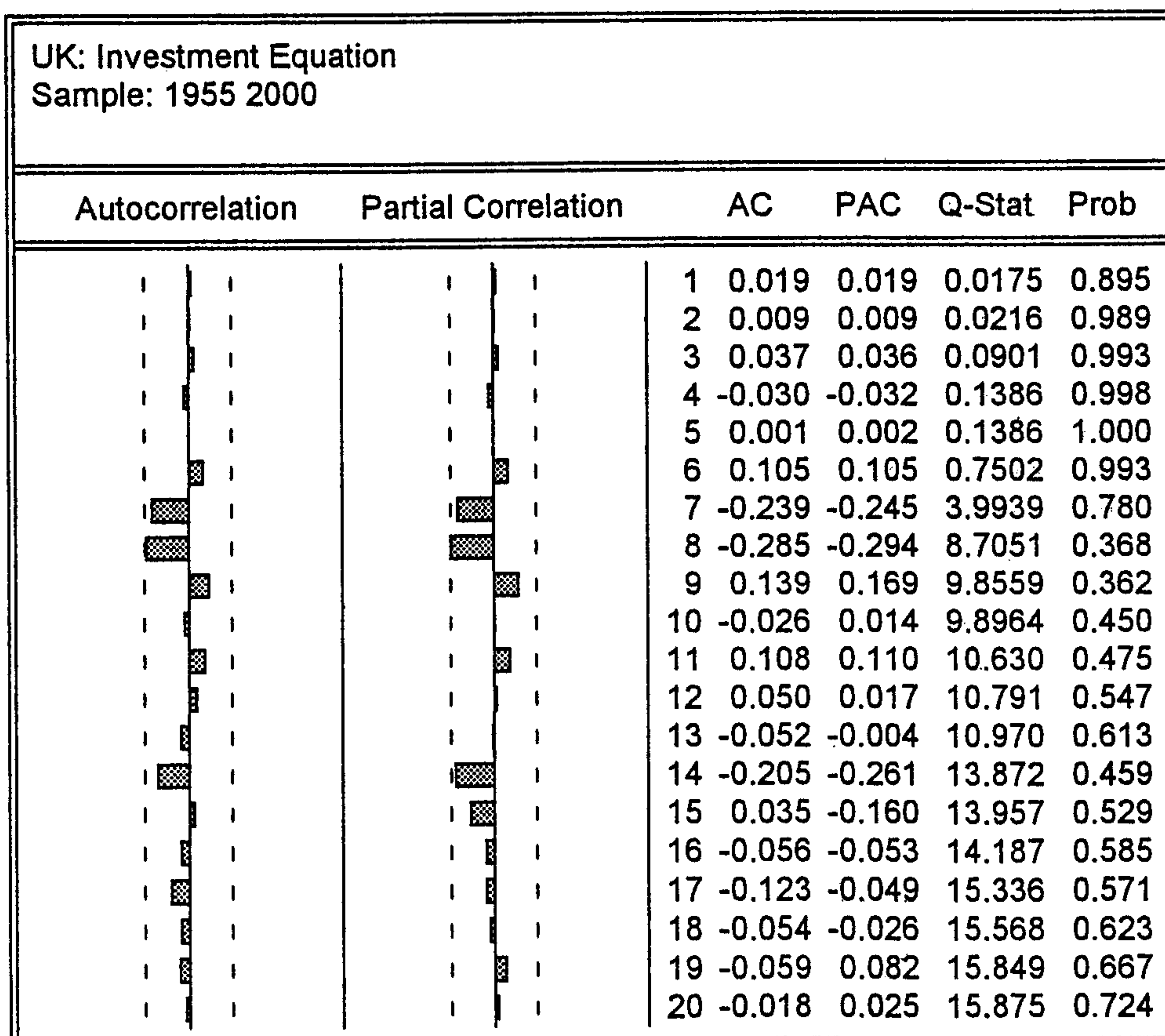
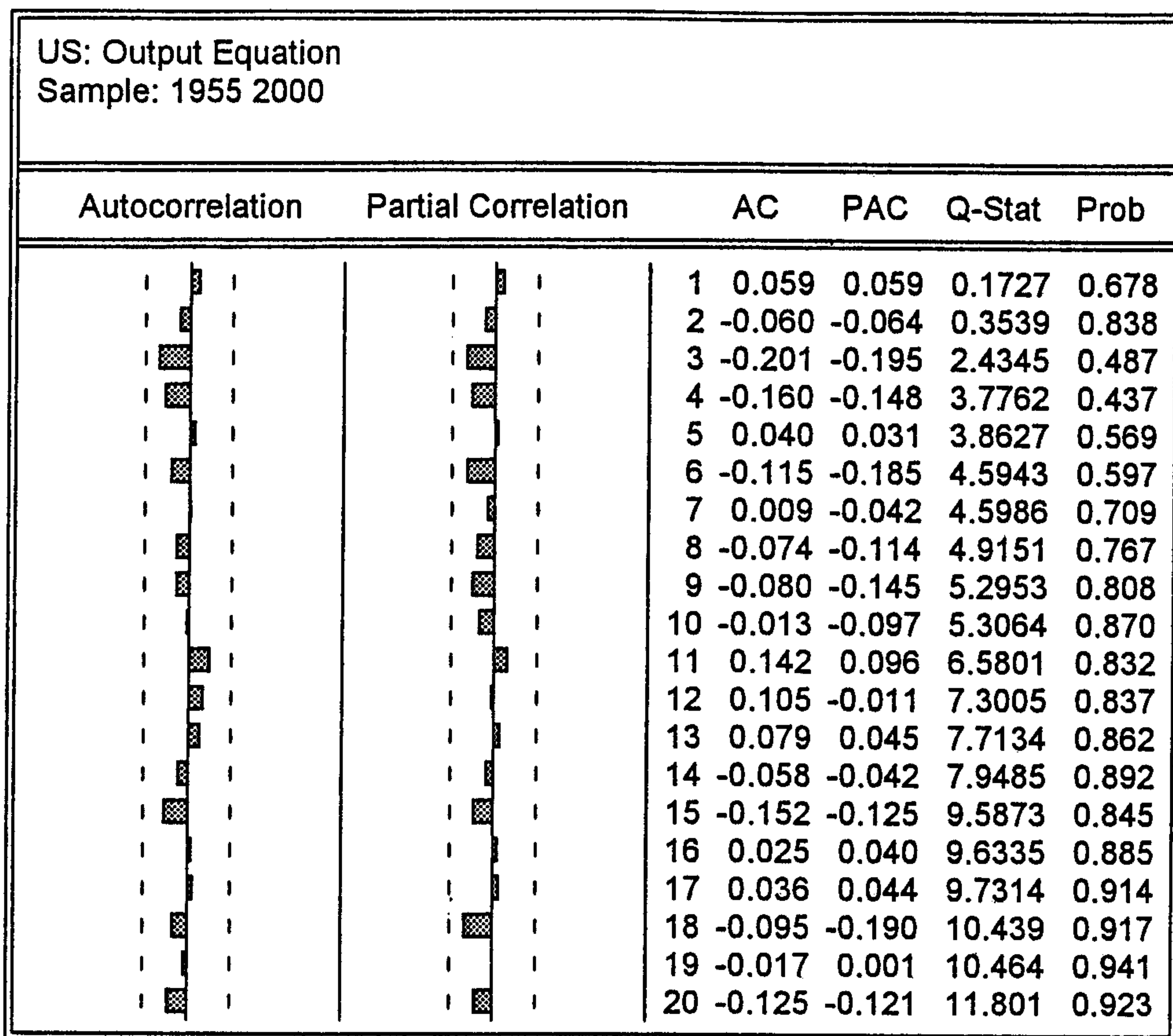


Figure A6: Analysis of Residuals (US)

Correlogram of Residuals



Correlogram of Residuals

