

Time-Varying Conditional Volume and Asset Pricing Relations

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Abstract

We use time-series properties of changes in share turnover with GMM tests to examine the number of factors that drive changes in conditional expected time-varying volume. In the three ten-year subperiods between 1966 and 1996, we detect no more than two factors. These findings support the hypothesis that changes in conditional time-varying expected volume can be represented parsimoniously as a function of a few factors. However, whether we detect one factor (as Tkac (1999) suggests) or two factors (as Lo and Wang (1998) indicates) depends on which sample period we use, on whether we sort portfolios by turnover or returns betas, and on whether we measure those betas relative to the equal- or the value-weighted index.

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1. Introduction

Recent studies by Lo and Wang (1998) and Tkac (1999) develop and test implications for share turnover of asset pricing models. Lo and Wang focus on separation theorems. In particular, they show that two-fund separation implies turnover will be identical for all securities while $(K+1)$ -fund separation implies turnover will satisfy an approximately linear K -factor structure. Empirically, Lo and Wang find that two principal components explain approximately 90 percent of the variation in turnover of portfolios ranked by turnover betas over each of six five-year periods. The three-fund separation these results suggest appears robust to changes in the index against which turnover betas are measured and to the use of differences in or levels of turnover to measure betas. These findings supplement returns-based tests in shedding light on which, if any, of the return generating processes implied by asset pricing models conform most closely to empirical predictions. The stability of the findings also suggests current price and volume relations may contain information about future price and volume behavior.

Tkac (1999) provides a theoretical rebalancing benchmark that connects trading of individual stocks to market-wide volume. Like Lo and Wang's (1998) two-fund separation, Tkac's result implies turnover will be identical for all securities if portfolio rebalancing is the sole motive for trade. To accommodate other motives, Tkac examines the idiosyncratic influences of institutional ownership, option availability, relative firm size, and membership in the S&P 500. Her findings support the hypotheses that both market-wide and firm-specific influences affect volume and that market adjustments for volume are important in filtering out anomalous trading.

On the surface, Lo and Wang's (1998) finding that *two* factors explain the cross-sectional variation in turnover seems to imply that Tkac's (1999) single-factor attempt to extricate market influences from volume may overstate the influence of idiosyncratic characteristics. On the other hand, Tkac's single-factor approach, if correct, calls Lo and Wang's second factor and three-fund separation result into question.

Of course, Lo and Wang's (1998) hypotheses, empirical methods, and data differ from those of Tkac (1999). To illustrate, Lo and Wang define turnover as the fraction of shares traded, while Tkac uses relative dollar volume. Lo and Wang use ~~only~~ *all* ordinary NYSE/AMEX common shares, while Tkac *only* uses all stocks in CRSP deciles 7 through 10. Lo and Wang's sample extends from July 1962 to December 1996, while Tkac's analysis extends from January 1988 through December 1991. Lo and Wang examine turnover across weekly intervals, while Tkac uses monthly observations. Such differences may invalidate direct comparisons between findings in Lo and Wang and Tkac. However, important empirical questions remain about the number of factors that *should* be used in tests such as Tkac's and about the number of separating portfolios that turnover data reveal.

This paper addresses some of those questions by using the time-series behavior of changes in turnover (as defined by Lo and Wang (1998)) to examine cross-sectional turnover models. Rouwenhorst (1999) demonstrates that turnover is positively related to the same attributes that drive cross-sectional differences in average returns. Other studies [e.g., Blume, Easley and O'Hara (1994), Conrad, Hameed, and Niden (1994) and Gervais, Kaniel, and Mingelgrin (1998)] suggest that volume is an important predictor of equity returns. Given these findings and those in numerous other studies [e.g., Gibbons

and Ferson (1985), Campbell (1987), Campbell and Hamao (1992), and Chang and Huang (1990)] that support time-varying conditional expected returns, we hypothesize that conditional expected changes in asset turnover could be time varying also. If that hypothesis is correct, Lo and Wang's K-fund separation theorem should be tested under the premise of time-varying turnover.

Accordingly, we first examine whether changes in asset turnover are partially predictable. We document of set of instruments that are useful in predicting changes in expected monthly stock turnover in US markets. Then, in the spirit of Gibbons and Ferson (1985) and Campbell (1987), we use GMM (Generalized Method of Moments) tests to infer the number of latent variables that drive changes in conditional expected turnover. If three-fund separation holds, as Lo and Wang (1998) suggest, we should reject a single-latent-variable model, but we should fail to reject a two-latent-variable model.

We also examine the consistency in the number of factors detected with volume and return data. If three-fund separation holds, we expect two latent variables to emerge with turnover and returns data. If inferences about future price and volume behavior are to be valid, we also expect the relations to be robust to whether portfolios are sorted by turnover or return betas (~~"need qualification" I am not sure what you mean here~~), [We can leave as it is. Originally, I was worried that in the GMM tests, we assume all portfolio beta coefficients are **not** time-varying. For turnover tests, if we sort portfolio by return beta, we may violate the assumption that the portfolio has constant turnover beta over the testing period. Our results do not show that this is an important concern. Let's

leave it as it is.] to the index against which betas are measured, and to the length of the test period.

Like Lo and Wang (1998), we use data from 1966 through 1996. Like Tkac (1999), we use monthly observations. *When portfolios are sorted by turnover betas, we find that the* The number of factors driving conditional expected *changes in turnover has been consistently less than two. The evidence renders strong support for Separation theorems in general. In addition to the sampling periods, the variation of the number of factors is also related* ~~—volume and returns in our analysis is small but unstable. It is also sensitive to~~ *the weighting scheme—whether turnover betas are measured relative to the equal or the value weighted index.* For the equal-weighted *portfolios* index, we find one latent variable in the first (1966-1976) and second subperiods (1977-1986), but two latent variables in the third (1987-1996). For the value-weighted *portfolios* index, one, two, and one factor(s) emerge(s) in the first, second, and third subperiods, respectively. *We obtain qualitatively the same results when turnover portfolios are sorted by return betas.*

For equally-weighted return portfolios, again, the number of hedge-portfolios identified in all three sub-periods has been consistently less than two. The results hold for both turnover-beta and return-beta sorted portfolios. In particular, for equally-weighted turnover-beta sorted portfolios, only one latent variable is identified for both turnover and return portfolios for the 1966-1976 and 1977-1986 subperiods. The joint results offer strong support for the two-fund separation theorem. Though the numbers of factors identified from turnover and return portfolios vary by one in the 1987-1996 sub-period, the results generally support a three-fund separation theorem.

We furnish evidence that the above results for turnover portfolios are insensitive to weighting scheme while return portfolios are. When turnover portfolios are sorted either by turnover-beta or return-beta, the number of identified latent variables has been consistently less than two. The results are consistent with Lo and Wang (1998) in some subperiods and with Tkac (1999) in others, depending on the index used to measure turnover and/or return betas. However, for the 1987-1996 subperiod, we have identified three (four) common factors for turnover-beta (return-beta) sorted value-weighted return portfolios while there is only one common factor driving changes in asset turnover.

The rest of the paper proceeds as follows. Section 2 motivates our methods. Section 3 discusses their technical details. Sections 4 and 5 present our data and empirical results, and Section 6 concludes.

~~Thus, the~~ only period in which the choice of index does not influence the results is between 1966 and 1976. Similar subperiod results hold for the number of factors driving conditional expected returns. On balance, therefore, our results are consistent with Lo and Wang (1998) in some subperiods and with Tkac (1999) in others, depending on the index used to measure turnover and/or return betas. (Well, see how this fits in after revision)

The rest of the paper proceeds as follows. Section 2 motivates our methods. Section 3 discusses their technical details. Sections 4 and 5 present our data and empirical results, and Section 6 concludes.

2. Methodology and Motivation for Time Series Methods

Asset pricing theories suggest that common movements characterize expected asset returns. Single-period models, such as the CAPM (Capital Asset Pricing Model) of Sharpe (1964) and the APT (Arbitrage Pricing Model) of Ross (1976), provide parsimonious representations in terms of factors of all excess returns. Initial tests of asset pricing relations use return data only. Like Lo and Wang (1998), initial researchers focus on cross-sectional relations. Black, Jensen, and Scholes (1972), Fama and MacBeth (1973), Gibbons (1982), and Roll and Ross (1980) provide early examples of this research. The first three studies test the one-factor CAPM; the fourth tests a multi-factor APT.

Like Lo and Wang, Roll and Ross use factor analysis. Several authors use this approach to discover the *number* of priced factors. Other authors attempt not only to count, but also to *identify and interpret* the factors. Because identification is difficult with factor analysis, later studies pre-specify multi-factor models in which the number *and* identity of the factors are fixed by researcher choice. Some of these studies [e.g., Chen, Roll, and Ross (1986)] use macroeconomic factors, while others [e.g., Fama and French (1992)] use factors based on firm-specific attributes. Though pre-specification solves the identity problem, it can not guarantee that relevant factors have not been omitted. Nor are included factors always tied tightly to equilibrium models of risk. Daniel and Titman (1997) show, for example, that returns on stocks are insensitive to their betas on market-to-book and size factor portfolios even though Fama and French (1992) argue that market-to-book ratios and market capitalization are important sources of security risk. Hence, interpretation remains difficult even when factors are pre-specified.

Besides the challenges mentioned above, the cross-sectional tests all presuppose that expected returns remain constant over the period of analysis. To circumvent this assumption, researchers have turned to time series tests. Early papers from this genre include Hansen and Hodrick (1983) and Gibbons and Ferson (1985). Gibbons and Ferson (p. 218) motivate their analysis by arguing that “. . . the underlying theory [of asset pricing] refers to moments *conditional on available information*. While these *conditional expectations may change over time*, empirical studies of asset pricing have not utilized this time series behavior when testing cross-sectional models of returns. (Emphasis added)” Gibbons and Ferson, and later Campbell (1987), show that under certain conditions latent variables tests based on predictable time-varying conditional expected returns can be construed as tests of general intertemporal asset pricing models.

Researchers have used latent-variables models to test the number of factors generating returns in the US stock [e.g., Gibbons and Ferson (1985) and Campbell (1987)], and bond [e.g., Chang and Huang (1990)] markets, the world price of covariance risk [e.g., Harvey (1991)], the integration of US and Japanese markets [e.g., Campbell and Hamao (1992)], and the regional integration of Asian equity markets [e.g., Chang, Pinegar, and Ravichandran (1994)]. Besides flexibility, these tests also nicely balance subjectivity and objectivity. The researcher selects the information variables that predict time varying expected returns, but the tests themselves determine the number of factors.

The greater congruity between the time series properties of returns and the methods used to test asset pricing is a nice feature of the latent variable approach. Despite this improvement, Elton (1999) argues against the use of realized returns *per se*. Specifically, he states (p. 1200) that “developing better measures of expected return and

alternative ways of testing asset pricing theories that do not require using realized returns have a much higher payoff than ... tests that continue to rely on realized returns as a proxy for expected returns.”

Thus, Lo and Wang (1998) and Tkac (1999) open new vistas with their analyses of turnover in an asset-pricing framework. Rouwenhorst (1999) also demonstrates that turnover is positively related to the same attributes that drive the cross-sectional differences in average returns. Other studies [e.g., Conrad, Hameed, and Niden (1994) and Gervais, Kaniel, and Mingelgrin (1998)] suggest that volume is an important predictor of equity returns. However, none of these studies considers the time-series properties of volume in discerning the number of factors that generate returns. If equity returns are time varying and can be predicted by volume, volume itself should vary through time in a predictable fashion. We confirm this assertion below and then use the predictive relations to discern the number of factors suggested by changes in volume.

3. Latent Variables Tests for Asset Turnover

Lo and Wang (1998) show that when the risk-free security, the market portfolio, and $K-1$ constant hedging portfolios are the relevant separating funds, the turnover of stock j at time t can be approximated by a linear K -factor model of the following form:

$$\tau_{jt} = F_{1t} + \sum_{k=2}^K S_{jk} F_{kt} + \mu_{jt} \quad j = 1, \dots, J \quad (1)$$

in which F_{1t} is turnover for the market portfolio from time $t-1$ to time t ; F_{kt} (for $k = 2, \dots, K$) are factors that depend on the turnover of the corresponding hedged portfolio k ; and S_{jk} is the number of shares of stock j held by fund k . Lo and Wang further demonstrate

that the approximation error of Equation (1) will be small when the amount of trading in the hedging portfolios is small. Note that Equation (1) can also be cast in terms of the change in turnover (τ_{jt}), and for reasons discussed below, using changes in turnover is better than using levels. Thus, we rewrite Equation (1) as

$$\tau_{jt} = \tau_{1t} + \sum_{k=2}^K S_{jk} \tau_{kt} + \mu_{jt} \quad j = 1, \dots, J \quad (1')$$

where τ_{jt} , τ_{1t} , and τ_{kt} refer, respectively, to the change in turnover of stock j , the market portfolio, and the hedged portfolio k ($k=2, \dots, K$) at time t .

Suppose that τ_{jt} is predictable using the information contained in Z_{t-1} , where Z_{t-1} is an $L \times 1$ vector of information variables: $(z_{1t-1}, \dots, z_{Lt-1})'$. We wish to test whether the time varying expected change in turnover is driven by a small number of unobservable latent variables. With Equation (1'), the number of latent variables identified should equal K . To proceed with the latent variables and GMM procedures for identifying the number of relevant factors that drive equation (1'), we express the predictability or the conditional expectation of τ_{jt} given Z_{t-1} as :

$$E(\tau_{jt}|Z_{t-1}) = \alpha_j' Z_{t-1} \quad (2)$$

Where $E(*|*)$ = conditional expectation operator,

τ_{jt} = the change in turnover volume of security j at time t , and

α_j = $(\alpha_{j1}, \alpha_{j2}, \dots, \alpha_{jL})' = (L \times 1)$ vector of coefficients.

Conditional expectations are assumed to be linear in the elements of Z_{t-1} . Rejection of the hypothesis that $\alpha_{jl} = 0$ (for $l = 1, \dots, L$) suggests the existence of information variables that forecast expected changes in turnover. We use the multifactor version of (1) to

examine if time-varying changes in turnover are governed by common factors. Within this framework, and with (2), equation (1') becomes:

$$E(\tau_{jt} | Z_{t-1}) = \sum_{k=1}^K \beta_{jk} \left(\sum_{l=1}^L \alpha_{kl} Z_{lt-1} \right) \quad (3)$$

The coefficients α_{kl} are projection coefficients of expected changes in turnover of hedge portfolios onto the information variables in the vector Z_{t-1} and β_{jk} is the constant regression coefficient corresponding to each factor k regarding changes in turnover for stock j in (1). Following Hansen and Hodrick (1983), we express the system of J regression equations describing J changes in turnover as:

$$\tau_t = AZ_{t-1} + \mu_t \quad (4)$$

where $\tau_t = (\tau_{1t}, \tau_{2t}, \dots, \tau_{Jt})'$

$A = (J \times L)$ matrix of coefficients with typical elements δ_{jl} and

$\mu_t = (\mu_{1t}, \mu_{2t}, \dots, \mu_{Jt})'$

Equation (4) implies the following testable restrictions,

$$\delta_{jl} = \sum_{k=1}^K \beta_{jk} \alpha_{kl} \quad (5)$$

where β_{jk} and α_{kl} are not directly observable, since changes in turnover of hedge portfolios are assumed to be unobservable. Hence, we standardize by setting $\beta_{jk} = 1$ for $j = k$ and $j, k \leq K$; and $\beta_{jk} = 0$ for $j \neq k$ and $j, k \leq K$. Thus, there are $JK - K^2$ free β 's and LK free α 's. There are a total of JL coefficients in the matrix, A , resulting in $(J-K)(L-K)$

overidentifying restrictions on the data. Technically, the model places overidentifying restrictions on the data if the number of latent variables K is strictly less than the number of portfolios J and the number of information variables L . In addition, practical considerations require that K be a small number so that a parsimonious representation of changes in turnover is feasible. A natural progression of the tests is to start with the single latent variable model and move up to a higher number of variables if the lower dimensions are rejected.

In testing the predictability of changes in turnover volume by information variables, we use the heteroskedasticity-consistent covariance estimators proposed by White (1980) to adjust for conditional heteroskedasticity. To estimate and test the latent variable model (equation (4)), we use Hansen's (1982) and Hansen and Singleton's (1982) generalized method of moments (GMM) nonlinear estimation technique that allows for conditional heteroskedasticity of an unknown nature.

We define a $(JL \times 1)$ vector function,

$$f(\tau_t, Z_{t-1}, \delta) = \mu_t \otimes Z_{t-1} \quad (6)$$

where \otimes denotes the Kronecker product. We further define a vector $g_T(\delta)$ containing the sample estimate for the mean corresponding to the elements of the vector function in (6) as :

$$g_T(\delta) = T^{-1} \sum_{t=1}^T f(\tau_t, Z_{t-1}, \delta) \quad (7)$$

which is evaluated at $\delta = \delta_0$. The GMM forms a vector of the orthogonality conditions, $g_T(\delta)$, and the parameter vector δ is chosen to make the orthogonality conditions as close to zero as possible by minimizing the quadratic form

$$L_T(\delta) = g_T(\delta)' W_T g_T(\delta) \quad (8)$$

Where W_T is a $(JL \times JL)$ symmetric, positive definite weighting matrix that defines the metric used to make g close to zero. Hansen (1982) provides the conditions under which minimization of $L_T(\delta)$ yields δ_T with the smallest asymptotic covariance matrix when

$$W_0 = E[f(\tau_t, Z_{t-1}, \delta) f(\tau_t, Z_{t-1}, \delta)']^{-1}. \quad (9)$$

Hansen (1982) shows that T times the minimized value of the quadratic form, $L_T(\delta)$, is asymptotically distributed as χ^2 with degrees of freedom equal to $(J-K)(L-K)$. This χ^2 statistic regarding the overidentifying restrictions provides a goodness of fit test for equation (5).

To implement the estimation procedure, we use a two-stage method suggested by Hansen and Singleton (1982). In the first stage, an arbitrary or suboptimal weighting matrix W_T is used to minimize $L_T(\delta)$ to get an initial consistent estimate of δ_T . These δ_T estimates are then used to construct the optimal weighting matrix W_T^* . In the second stage, the optimal W_T^* is used to minimize (8) to obtain a second-stage estimate of δ_T^* . Although a two-stage procedure is asymptotically efficient, Ferson and Foerster (1994) show that the finite sample performance of GMM is improved when further iterations are conducted to minimize the dependence of the estimator on the initial weighting matrix. Hence, if necessary, we extend the two-stage process.

4. Data Description

Panels A and B of Table 1 summarize turnover and return data for nine double-sorted portfolios of AMEX/NYSE stocks from the Center for Research in Securities Prices (CRSP) data base. For each month from January 1962 through December 1996, we sort

securities into three size-ranked portfolios according to their previous year-end market capitalization.¹ Each size portfolio is then further partitioned into three portfolios according to the turnover betas of its component securities. The turnover beta of security j (β_j) for month t is estimated using data for the previous 36 months (i.e. from $t-36$ to $t-1$) according to equation (10):

$$\tau_{jt} = \alpha_j + \beta_j \tau_{mt} + \varepsilon_{jt} \quad (10)$$

in which τ_{jt} (τ_{mt}) is the turnover of security j (the market portfolio) at time t . The double sorting process produces nine size-turnover-beta-sorted portfolios. We measure turnover betas relative to an equal-weighted and a value-weighted index of all AMEX/NYSE stocks, but we report results in tables only for the equal-weighted index.

Portfolios 1 through 3, 4 through 6, and 7 through 9 contain the small-, mid-, and large-cap firms in our sample. Within each size-sorted third, turnover betas increase with the portfolio number. For the small-cap portfolios, for example, firms in portfolio 1 have the lowest turnover betas, firms in portfolio 2 have the next lowest betas, and firms in portfolio 3 have the highest betas.

Turnover betas range from .539 for portfolio 1 to 1.291 for portfolio 9. Though small firms have lower turnover betas, on average, the relation is not monotonic. The mean beta for portfolio 3 (.915) is higher than the mean betas for portfolios 4 (.632) and 5 (.876), and the mean beta for portfolio 6 (1.236) is higher than the mean betas for portfolios 7 (.798) and 8 (.933). Within size categories, high-beta stocks do not always

¹ Securities with an initial public offering one year are included in the sample the next year. Because the CRSP tapes occasionally contain errors, with daily turnover rates exceeding 1,000% in some cases, we also exclude securities with daily turnover exceeding 50%. This threshold is arbitrary, but CRSP data providers confirm that there are errors in some of the volume estimates. Because we are unsure whether all

have the largest mean capitalization. However, the low-, medium-, and high-beta estimates for stocks in the small-cap portfolios are all lower than the corresponding estimates for stocks in the mid-cap portfolios which, in turn, are smaller than the corresponding estimates for stocks in the large-cap portfolios. Mean turnover estimates exhibit patterns similar to, but not as pronounced as, the patterns exhibited by the turnover betas. Within size groups mean turnover increases as turnover betas increase, but across groups the high-beta large-cap stocks in portfolio 9 have a lower mean turnover (.0555) than the high-beta mid-cap stocks in portfolio 6 (.0651).

As expected, autocorrelation coefficients are high for turnover even with a 12-month lag. They also tend to be higher for large-cap stocks at all lags. The persistence in turnover suggests some form of nonstationarity. To avoid spurious predictive relations and to perform more meaningful statistical inference, we first difference turnover in our tests below. More sophisticated methods [e.g., Gallant, Rossi, and Tauchen (1994)] are available for detrending the data, but Lo and Wang (1998) show that there are costs to each approach. Thus, we employ a simple first difference to facilitate interpretation.

Panel B summarizes return data for the same portfolios. Instead of turnover betas, the panel reports return betas, but the second portfolio sort is still based on turnover betas. Within each size-sorted third of the sample, return betas increase with turnover betas. However, across all nine portfolios the correlation (.28) between mean turnover betas in panel A and mean return betas in panel B is not strong. Thus, examining the sensitivity of our results to sorting criteria may be important. ~~{come back here?}~~

errors have been corrected, we use the 50% cutoff rate. Lo and Wang (1998) also had to use a filter in their analysis.

Compared with the autocorrelation coefficients for turnover, the coefficients for returns are small. Nevertheless, portfolios 1 through 6 all have significant one-month lags. Potential causes of this (cross-)autocorrelation are discussed by Lo and MacKinlay (1990), McQueen, Pinegar, and Thorley (1996), and others at length. The significant 12-month lag for portfolios 1 through 3 is consistent with the January effect discussed by Rozeff and Kinney (1976), Reinganum (1981), Keim (1983), and others. Holding firm size constant, neither the one-month lag, nor the 12-month lag appears to change significantly with changes in the turnover betas.

Table 2 reports summary statistics for the instrumental variables we use to predict changes in turnover. They include the lagged [first](#) difference in turnover for the market as a whole (DMAVTO), the lagged cross-sectional dispersion of turnover for the market as a whole (MDPTO), the lagged average market return (MAVRE), the lagged dispersion in returns for the market (MDPRE), and a January dummy variable (DJAN). [The dispersion estimates for returns and turnover are cross-sectional standard deviations for the respective variables.](#) Except DJAN, most of these instruments are significantly positively correlated with each other. Thus, to some extent, they may measure the same thing. However, even the highest correlation (0.421, between MAVRE and MDPRE) indicates that collinearity problems should be small. Thus, we now proceed to predict changes in time-varying turnover.

5. Empirical Results

5.1 Base Case

We begin our analysis in this section by presenting results for tests of the number of factors driving conditional expected changes in turnover and returns when portfolios are

formed according to procedures outlined above. Then, we present robustness tests of the base case results. Our first step is to establish predictive relations between the change in turnover and our instrumental variables. Table 3 reports results of the following regression for each portfolio $j = 1, \dots, 9$

$$\Delta t_{j,t} = \mathbf{a}_{j,0} + \mathbf{a}_{j,1} * DMAVTO_{t-1} + \mathbf{a}_{j,2} * MDPTO_{t-1} + \mathbf{a}_{j,3} * MAVRE_{t-1} + \mathbf{a}_{j,4} * MDPRE_{t-1} + \mathbf{a}_{j,5} * DJAN_t + \mathbf{e}_{j,t} \quad (11)$$

in which $\Delta t_{j,t}$ [Please check notations here] is a equal-weighted average of the change in turnover at time t for portfolio j , and the instrumental variables are as defined above.

Coefficients in (11) that are significant at the .05 (.10) level are marked with double (single) asterisk. The coefficients on $DMAVTO_{t-1}$ and $MAVRE_{t-1}$ are significant at the .05 level for each of the nine portfolios. The coefficients on $DJAN_t$ are significant for all but portfolio 3, and the coefficients on $MDPTO_{t-1}$ and $MDPRE_{t-1}$ are significant for at least five of the nine portfolios. $MDPTO_{t-1}$ has more predictive power for small-cap firms, while $MDPRE_{t-1}$ has more predictive power for large-cap firms. In general, the change in turnover is more predictable for large-cap than for small-cap firms. However, the adjusted R^2 's in Table 3 (.105 to .272) clearly show that changes in turnover are predictable for all nine portfolios.

Given this evidence of predictability, we now use Hansen's (1982) and Hansen and Singleton's (1982) generalized method of moments (GMM) nonlinear estimation technique to determine the number of factors driving time-varying changes in conditional expected turnover. We focus on results for the three subperiods from March 1966 through December 1976, from January 1977 through December 1986, and from January 1987 through December 1996. The reasons for our subperiod analysis are twofold. First, the

nature of the market has changed dramatically during our sample period. Many new securities have been introduced, index funds have become popular, and portfolio insurance and dynamic trading strategies have become available. Second, Lo and Wang (1998) also focus on subperiods. However, they use weekly observations over five-year subperiods; whereas, we use monthly observations over 10-year subperiods

Results of our subperiod analysis are reported in Table 4. In the first and second subperiods, we fail to reject the hypothesis that a single latent variable drives changes in conditional expected turnover. In the third subperiod, we reject a one-factor model, but fail to reject a two-factor model. Thus, our first two subperiods are consistent with Tkac (1999), while our third subperiod is consistent with Lo and Wang (1998).

Differences in the number of factors revealed by turnover data in our tests vis-à-vis those of Lo and Wang illustrate the sensitivity of the results to methodological changes. They may also illustrate difficulties in interpretation. For example, Lo and Wang's first factor in their Table 8 explains between 77.2% and 82.6% of the variation in changes in weekly turnover across sample periods. The second component explains between 5.0% and 8.0%. If the second component is economically significant, however, the third may be also because it explains between 3.0% and 4.8% of the variation in first differences in weekly turnover. In three of the six subperiods, the third component is at least 70% as large as the second. The interpretation problem arises, of course, because no rule defines when one factor is economically significant and another is not. Nor can statistical significance resolve the issue in Lo and Wang's analysis because the asymptotic standard errors of the standardized principal components are computed under

the assumption of IID Gaussian data. As Lo and Wang (p. 34) warn, this assumption is "hardly appropriate for weekly US stock returns and even less convincing for turnover."

What statistical properties conditional changes in turnover exhibit is unknown. However, Table 3 presents strong evidence that changes in turnover vary over time in a predictable manner, and Table 4 shows that the number of factors emerging from the predictive relations is unstable across subperiods. Subject to the constraint that our tests are appropriate for the distributional properties of conditional changes in turnover, they reveal one factor in the first and second subperiods and two factors in the third. In general, then, the results in Table 4 are consistent with the spirit of Lo and Wang (1998) and Tkac (1999) in that they suggest either two- or three-factor separation. We now examine whether similar findings hold when we use return vis-à-vis turnover data.

As before, the first step is to predict time-varying returns for our equal-weighted portfolios. Many authors (e.g., Campbell (1987), Campbell and Hamao (1992), and Chang and Huang (1990)) demonstrate that stock and bond returns can be predicted using various instruments. Consequently, we use those instruments shown to be successful in previous studies in the following regression

$$\begin{aligned}
 PRET_{j,t} = & \mathbf{a}_{j,0} + \mathbf{a}_{j,1} * DPR_{t-1} + \mathbf{a}_{j,2} * LTMST_{t-1} + \mathbf{a}_{j,3} * RSR_{t-1} + \\
 & \mathbf{a}_{j,4} * DMAVTO_{t-1} + \mathbf{a}_{j,5} * DJAN_t + \mathbf{e}_{j,t}
 \end{aligned} \tag{12}$$

The last two variables in (12), $DMAVTO_{t-1}$ and $DJAN_t$, were also used to predict changes in turnover. Besides those variables, regression (12) includes the lagged dividend yield on the market (DPR_{t-1}), the lagged spread between returns on the 20 year government bond and the 30-day T-bill ($LTMST_{t-1}$), and the lagged relative short rate

(RSR_{t-1}), i.e., the 30-day T-bill total return minus the moving average total return on 30-day T-bills over the last year.

Though the instruments in (12) differ from those in (11), we use exactly the same stocks, portfolio weights, and sorting procedures to predict returns as we used to predict changes in turnover. Nevertheless, adjusted R^2 's in Table 5 (.024 to .182) are noticeably lower than adjusted R^2 's in Table 3 (.105 to .272). In general, these adjusted R^2 's indicate that time-varying turnover is more predictable for large-cap firms; whereas, time-varying returns are more predictable for small-cap firms. Nevertheless, returns are predictable for all portfolios in Table 5. The coefficients on $DJAN_t$ are significant at the .05 level for all nine portfolios and the coefficients on $LTMST_{t-1}$ are significant at the .10 level for seven portfolios.

Table 6 reports GMM tests of the number of latent variables driving time-varying conditional expected returns. In each subperiod, we fail to reject the hypothesis that only one latent variable is present. In Table 4, only one factor is present in the first and the second subperiods, but two factors emerge in the third. Therefore, the results generally support a parsimonious representation of factors common to all expected returns or to all expected changes in turnover. In the first and second periods, both time-varying expected returns and time-varying expected changes in turnover are driven by only one factor.

Therefore, both results support the two-fund separation theorem. In the third subperiod, the number of factors driving conditional expected changes in volume (2) exceeds the number of factors driving conditional expected returns (1) by one, but both numbers are small.

Figure 1 summarizes Tables 4 and 6 graphically and shows the similarity in the numbers of factors across subperiods with conditional changes in expected turnover and conditional expected returns. As Elton (1999) predicted, using data other than realized returns can enrich our understanding of asset pricing theorems. Interestingly, however, the number of factors detected with *time-varying expected* ~~realized~~ returns is generally consistent with the number of factors detected with *time-varying expected* changes in turnover. Therefore, we now examine the robustness of this result.

5.2 Robustness Tests

Our first robustness test repeats the analysis above with a different sorting procedure. Specifically, after sorting by size, we sort by return rather than turnover betas. As before, we measure betas relative to an equal-weighted index of AMEX/NYSE stocks. Panel A of Table 7 contains results for conditional changes in expected turnover; panel B reports results for conditional expected returns. Figure 2 summarizes the results of both panels.

Only one latent variable drives conditional changes in expected turnover across all subperiods in Table 7. However, two latent variables emerge in the second and third subperiods for conditional expected returns. Relative to the base case, then, the number of factors driving conditional changes in expected turnover declines by one in the last subperiod when portfolios are sorted by return, rather than turnover, betas. The number of factors driving conditional expected returns, on the other hand, increases by one in the second and third subperiods. Except for the last subperiod, the results based on

conditional changes in expected turnover and conditional expected returns are consistent with two- or three-fund separation.

Our second robustness test uses the original sorting procedure, but measures turnover betas relative to the value-weighted AMEX/NYSE index. For brevity, we summarize the results in Figure 3 without reporting detailed information in a separate table. Compared with the base case, Figure 3 indicates one more (fewer) factor in the second (third) subperiod for changes in conditional expected turnover. It indicates two more factors for the third subperiod with conditional expected returns. As before, the only subperiod that produces results that are inconsistent with two- or three-fund separation is the third subperiod.

Our final subperiod robustness test combines the alterations in the previous two sensitivity analyses. Thus, we sort by return betas measured against the value-weighted AMEX/NYSE index. The results, summarized in Figure 4, show that the numbers of latent variables emerging with conditional changes in expected turnover (1, 2, 1) are the same as in Figure 3. However, the numbers of factors driving conditional expected returns are (1, 2, 4) increase by one in the second and third subperiods. In general, the number of factors detected in tests with conditional expected returns is more sensitive to changes in the weighting and/or sorting scheme than is the number of factors detected with conditional changes in expected turnover. The relative stability with conditional changes in expected turnover is consistent with the findings of Lo and Wang (1998).

[Suggesting to eliminate this section. Note that even in early Fama and Macbeth (1973) paper, CAPM is tested by sub-period. It is also true for Chen, Roll and Ross (1986). This is also why Lo and Wang (1998) use sub period] Our final robustness test

examines the number of factors that are detected when the sample is not partitioned into 10-year subperiods. Lo and Wang (1998) do not present total sample results. Therefore, we do not know the number of factors their analysis would reveal over the total sample period. Nevertheless, we perform total sample tests for completeness' sake. In those tests, we reject even a four-factor model with both turnover and returns data when turnover betas are measured relative to the equal-weighted index. By definition, our analysis includes (J-K)(L-K) overidentifying restrictions, for J (= 9) portfolios and L (= 6) instrumental variables. Thus, our rejection of a four-factor model raises questions about the premise that asset-pricing relations can be expressed parsimoniously. This finding supports Bessembinder, Chan, and Seguin (1996) and suggests that the more disparate the firms and the longer the time period, the greater the likelihood that volume will change idiosyncratically and/or that the number of factors that drive conditional changes in expected turnover will change intertemporally. Hence, the chance that factor model tests will detect a small number of *common* factors is reduced.

Despite these differences, our results are broadly consistent with those assumed and/or reported by Tkac (1999) and Lo and Wang (1998) in that we do detect a small number of factors during each of the subperiods. This finding supports the hypothesis that common factors drive volume. However, whether there is one factor (as Tkac assumes) or two factors (as Lo and Wang's analysis indicates) depends on which sample period we use, on whether we use turnover or returns betas, and on whether we measure those betas relative to the equal- or the value-weighted index.

6. Conclusion

We use time-series properties of changes in share turnover with GMM tests to examine the number of factors that drive changes in conditional expected time-varying volume. In the three ten-year subperiods between 1966 and 1996, our tests detect no more than two factors. These findings support recent evidence in Lo and Wang (1998) and Tkac (1999). Similar results obtain when we use the identical portfolios and sorting methods to detect the number of factors driving conditional expected returns. However, the number of factors emerging from our tests is sensitive to whether we sort by turnover or returns betas and to whether we measure those betas relative to an equal- or a value-weighted index. Despite these sensitivities, our evidence is generally consistent with the hypothesis that changes in conditional expected turnover and conditional expected returns can be expressed parsimoniously as a function of a few latent variables.

References

- Bessembinder, H., K. Chang, and P. Seguin, 1996, "An Empirical Examination of Information, Differences of Opinion, and Trading Activity," *Journal of Financial Economics* 40, 105-134.
- Black, F., M. Jensen, and M. Scholes, 1972, "The Capital Asset Pricing Model: Some Empirical Tests," in M. Jensen ed. *Studies in the Theory of Capital Markets* (Praeger: New York) 79-121.
- Blume, L., D. Easley, and M. O'Hara, 1994, "Market Statistics and Technical Analysis: The Role of Volume," *Journal of Finance* 49, 153-181.
- Campbell, J., 1987, "Stock Returns and the Term Structure," *Journal of Financial Economics* 18, 373-399.
- Campbell, J. and Y. Hamao, 1992, "Predictable Stock Returns in the United States and Japan: A Study of long-term Capital Market Integration," *Journal of Finance* 47, 43-69.
- Campbell, J., A. Lo and A. MacKinlay, 1997, *The Econometrics of Financial Markets*, Princeton University Press, Princeton, New Jersey.
- Chang, E. C. and R. D. Huang, 1990, "Time-Varying Return and the Risk in the Corporate Bond Market," *Journal of Financial and Quantitative Analysis* 25, 323-340.
- Chang, E. C., J. M. Pinegar, and R. Ravichandran, 1994, "Predictability and Regional Integration of Pacific Basin Equity Markets," *Journal of International Financial Management and Accounting* 5, 25-46.
- Chen, Nai-Fu, R. Roll, and S. A. Ross, 1986, "Economic Forces and the Stock Market," *Journal of Business* 59, 383-403.
- Conrad, J., A. Hameed, and C. Niden, 1994, "Volume and Auto-Covariances in Short-Horizon Individual Security Returns," *Journal of Finance* 49, 1305-1329.
- Daniel, K., and S. Titman, 1997, "Evidence on the Characteristics of Cross-Sectional Variation in Stock Returns," *Journal of Finance* 52, 1-33.
- Elton, E. J., 1999, "Expected Return, Realized Return, and Asset Pricing Tests," *Journal of Finance* 54, 1199-1220.
- Fama, E., and K. R. French, 1992, "The Cross-Section of Expected Stock Returns," *Journal of Finance* 47, 427-465.
- Fama, E., and J. MacBeth, 1973, "Risk, Return, and Equilibrium: Empirical Tests," *Journal of Political Economy* 81, 607-636.

Ferson, W. and S. Foerster, 1994, "Finite Sample Properties of the Generalized Method of Moments in Tests of Conditional Asset Pricing Models," *Journal of Financial Economics* 36, 29-55.

Gallant, R., P. Rossi, and G. Tauchen, 1994, "Stock Prices and Volume," *Review of Financial Studies* 5, 199-242.

Gervais, S., R. Kaniel, and D. Mingelgrin, 1998, "The High Volume Return Premium," working paper, Wharton.

Gibbons, M. R. and W. Ferson, 1985, "Testing asset Pricing Models with Changing Expectations and an Unobservable Market," *Journal of Financial Economics* 14, 217-236.

Gibbons, M. R., 1982, "Multivariate Tests of Financial Models: A New Approach," *Journal of Financial Economics* 10, 3-27.

Hansen, L., 1982, "Large Sample Properties of Generalized Method of Moments Estimators," *Econometrica* 50, 1029-1054.

Hansen, L. and R. Hodrick, 1983, "Risk Averse Speculation in the Forward Foreign Exchange Market: An Econometric Analysis of Linear Models," in J. Frenkel (ed.), *Exchange Rates and International Macroeconomics*, University of Chicago Press, Chicago, Illinois.

Hansen, L. and K. Singleton, 1982, "Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models," *Econometrica* 50, 1269-1288.

Harvey, C.R., 1991, "The World Price of Covariance Risk," *Journal of Finance* 46, 111-157.

Keim, D., 1983, "Size Related Anomalies and Stock Return Seasonalities: Further Empirical Evidence," *Journal of Financial Economics* 12, 13-32.

Lo, A. and C. MacKinlay, 1990, "When are Contrarian Profits due to Stock Market Overreaction?" *Review of Financial Studies* 3, 175-205.

Lo, A. and J. Wang, 1998, "Trading Volume: Definitions, Data Analysis, and Implications of Portfolio Theory," Working Paper, Sloan School of Management, MIT.

McQueen, G, M. Pinegar, and S. Thorley, 1996, "Delayed Reaction to Good News and the Cross-Autocorrelation of Portfolio Returns," *Journal of Finance* 51, 889-919.

Newey, W. and K. West, 1987, "A Simple, Positive Semi-Definite, Heteroscedasticity and Autocorrelation Consistent Covariance Matrix," *Econometrica* 55, 703-708.

Reinganum, M., 1981, "Misspecification of Capital Asset Pricing: Empirical Anomalies Based on Earnings Yields and Market Values," *Journal of Financial Economics* 9, 19-46.

Roll, R. and S. A. Ross, 1980, "An Empirical Investigation of the Arbitrage Pricing Theory," *Journal of Finance* 35, 1073-1103.

Rouwenhorst, K. G., 1999, "Local Return Factors and Turnover in Emerging Stock Markets," *Journal of Finance* 54, 1439-1464.

Rozeff, M. and W. Kinney, 1976, "Capital Market Seasonality: The Case of Stock Returns," *Journal of Financial Economics* 3, 379-402.

Tkac, P. A., 1999, "A Trading Volume Benchmark: Theory and Evidence," *Journal of Financial and Quantitative Analysis* 34, 89-114.

White, H. 1980, "A Heteroskedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity," *Econometrica* 48, 817-838.

Figure 1
Differences in the Number of Factors Driving Conditional Changes in Expected Turnover and Returns for Turnover-Beta-Sorted Portfolios when Betas are Measured Relative to an Equal-Weighted Index

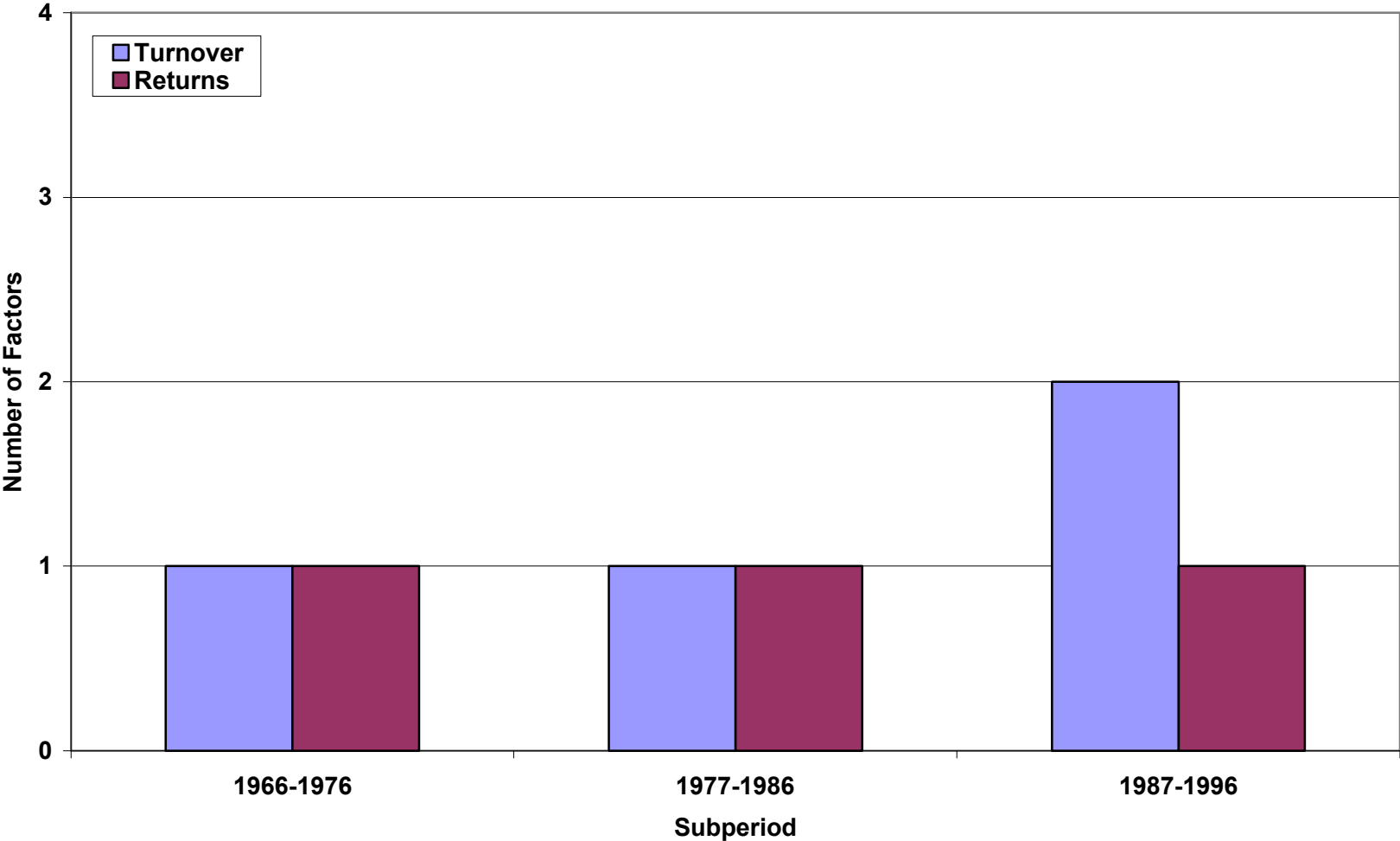


Figure 2
Differences in the Number of Factors Driving Conditional Changes in Expected Turnover and Returns for Returns-Beta-Sorted Portfolios when Betas are Measured Relative to an Equal-Weighted Index

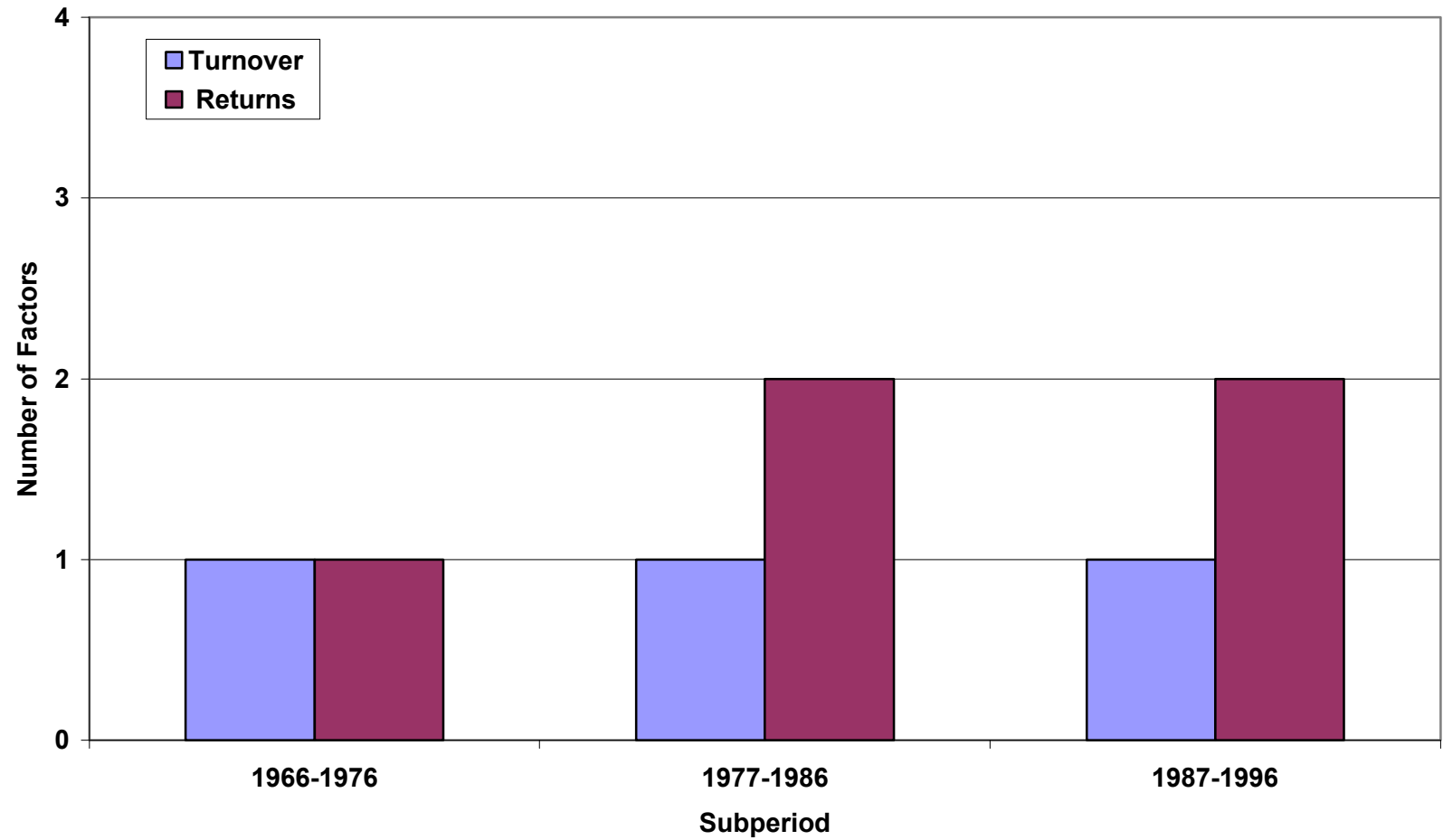


Figure 3
Differences in the Number of Factors Driving Conditional Changes in Expected Turnover and Returns for Turnover-Beta-Sorted Portfolios when Betas are Measured Relative to a Value-Weighted Index

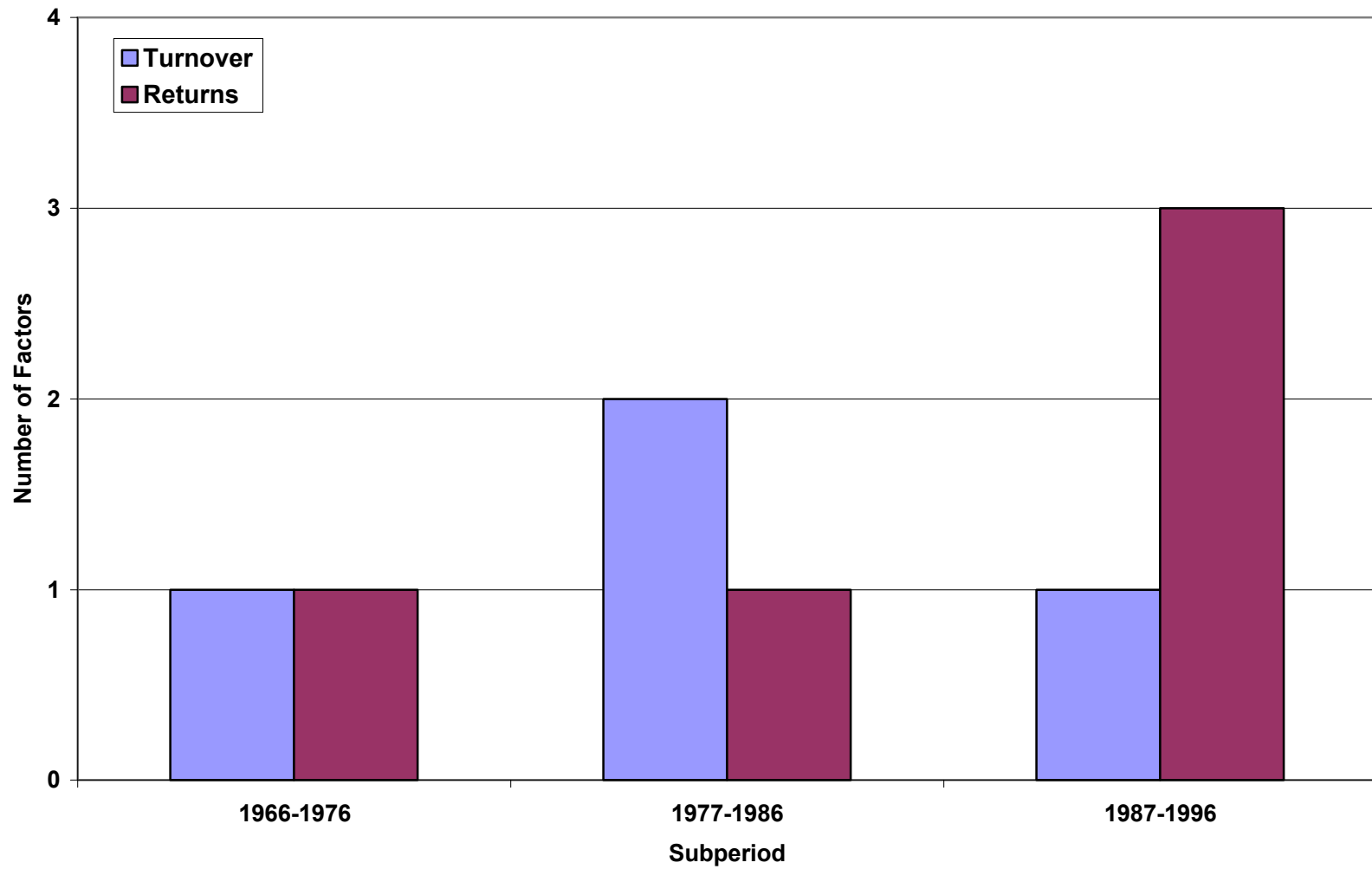


Figure 4
Differences in the Number of Factors Driving Conditional Changes in Expected Turnover and Returns
for Returns-Beta-Sorted Portfolios when Betas are Measured Relative to a Value-Weighted Index

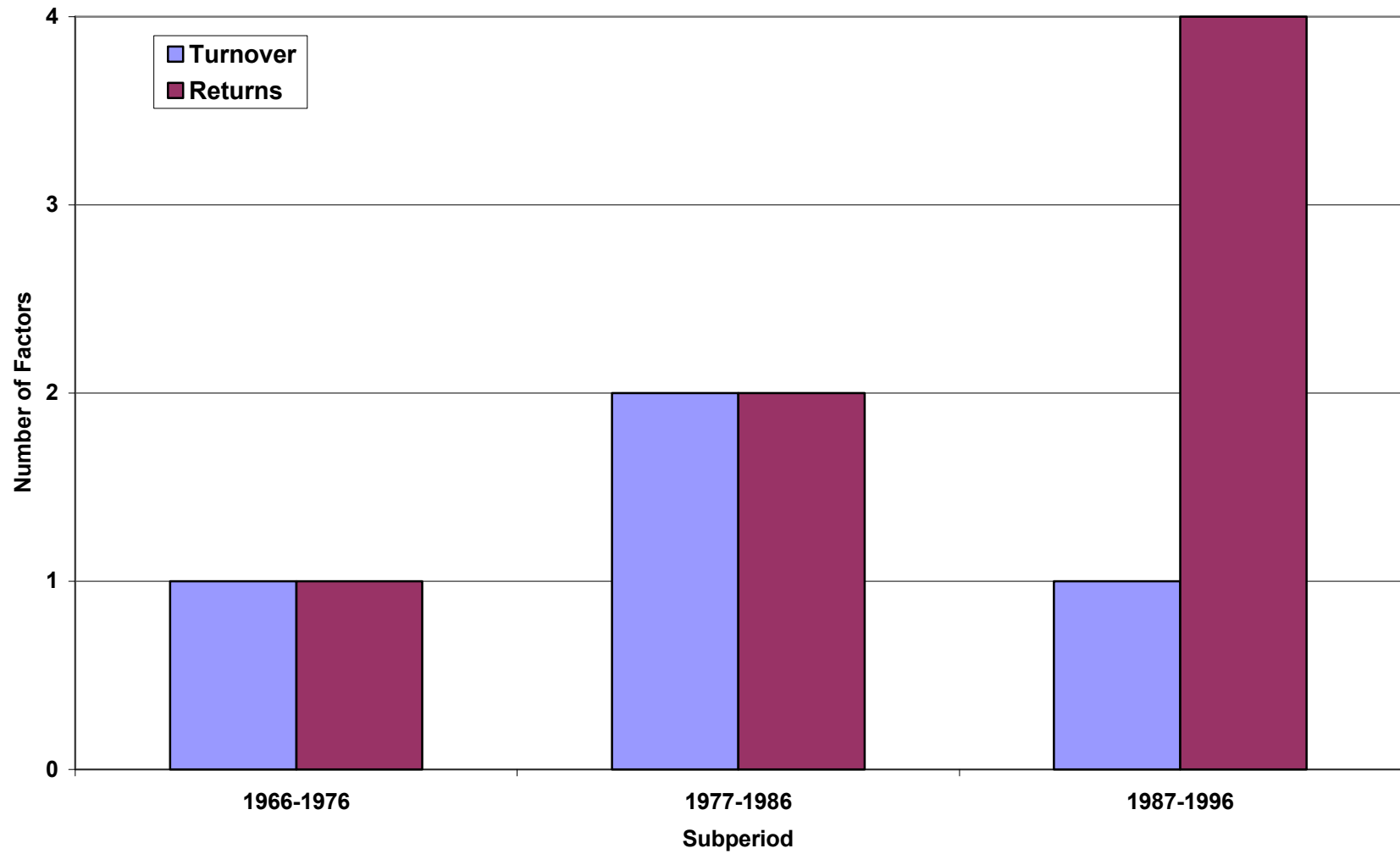


Table 1
Summary Statistics for Turnover and Returns of Size- and Turnover-Ranked Portfolios
Of AMEX / NYSE Equal-weighted (1966-1996)

Panel A	(10 ⁶ US\$) Average Capitalization	Volume Beta	Monthly Turnover				Autocorrelation Coefficients				
			Mean	Std. Dev.	Max	Min	ρ_1	ρ_2	ρ_3	ρ_6	ρ_{12}
Portfolio 1	78.747	0.539	0.0313	0.0122	0.0824	0.0081	0.793	0.666	0.582	0.489	0.379
Portfolio 2	86.244	0.712	0.0406	0.0183	0.1311	0.0094	0.825	0.713	0.648	0.502	0.410
Portfolio 3	76.553	0.915	0.0498	0.0231	0.1596	0.0120	0.811	0.675	0.602	0.490	0.435
Portfolio 4	586.194	0.632	0.0302	0.0125	0.0705	0.0090	0.898	0.877	0.845	0.776	0.697
Portfolio 5	610.273	0.876	0.0423	0.0163	0.0972	0.0108	0.867	0.825	0.790	0.715	0.660
Portfolio 6	570.136	1.236	0.0651	0.0227	0.1452	0.0166	0.819	0.764	0.711	0.609	0.628
Portfolio 7	10798.808	0.798	0.0317	0.0184	0.0900	0.0063	0.950	0.945	0.933	0.902	0.848
Portfolio 8	8867.496	0.933	0.0383	0.0202	0.1016	0.0095	0.933	0.927	0.913	0.881	0.839
Portfolio 9	6125.215	1.291	0.0555	0.0251	0.1449	0.0178	0.901	0.900	0.884	0.844	0.821

Panel B	(10 ⁶ US\$) Average Capitalization	Return Beta	Monthly Return				Autocorrelation Coefficients				
			Mean	Std. Dev.	Max	Min	ρ_1	ρ_2	ρ_3	ρ_6	ρ_{12}
Portfolio 1	78.747	0.962	0.1661	0.0592	0.3157	-0.2631	0.203	-0.020	-0.018	-0.012	0.237
Portfolio 2	86.244	1.202	0.0165	0.0733	0.4278	-0.3209	0.193	-0.032	-0.030	-0.009	0.216
Portfolio 3	76.553	1.414	0.0169	0.0871	0.4924	-0.3299	0.198	-0.011	-0.043	-0.021	0.214
Portfolio 4	586.194	0.720	0.0122	0.0442	0.2558	-0.2346	0.153	-0.044	-0.015	0.000	0.112
Portfolio 5	610.273	0.993	0.0124	0.0593	0.2846	-0.2941	0.145	-0.033	-0.038	-0.013	0.073
Portfolio 6	570.136	1.236	0.0110	0.0741	0.3488	-0.3355	0.149	-0.032	-0.049	-0.026	0.072
Portfolio 7	10798.808	0.532	0.0102	0.0389	0.1740	-0.1598	0.042	-0.051	0.015	-0.045	0.055
Portfolio 8	8867.496	0.738	0.0108	0.0483	0.1870	-0.2478	0.086	-0.055	-0.033	-0.023	0.009
Portfolio 9	6125.215	0.973	0.0098	0.0614	0.2362	-0.2885	0.104	-0.020	-0.043	-0.039	-0.007

Table 2
Summary Statistics for Instrumental Variables Used to Predict Changes in Turnover for
Size- and Turnover-Beta Sorted AMEX / NYSE Stocks (1966-1996)

	DMAVTO	MDPTO	MAVRE	MDPRE	DJAN
Mean	0.0001	0.0032	0.0138	0.1445	N/A
Std. Dev.	0.0080	0.0023	0.0587	0.0099	N/A
Max	0.0327	0.0138	0.3098	0.1261	N/A
Min	-0.0410	0.0002	-0.2784	0.0051	N/A
<i>Correlations</i>					
DMAVTO	1.000				
MDPTO	0.245	1.000			
MAVRE	0.302	0.155	1.000		
MDPRE	0.289	0.269	0.421	1.000	
DJAN	0.170	-0.061	0.022	-0.017	1.000

Note: The instruments include the lagged difference in turnover for the market as a whole (DMAVTO), the lagged cross-sectional dispersion of turnover for the market as a whole (MDPTO), the lagged average market return (MAVRE), the lagged dispersion in returns for the market (MDPRE), and a January dummy variable (DJAN).

Table 3

**Regressions of Changes in Turnover on Instrumental Variables
over the Period 1966 to 1996**

$$\Delta t_{j,t} = \alpha_{j,0} + \alpha_{j,1} * DMAVTO_{t-1} + \alpha_{j,2} * MDPTO_{t-1} + \alpha_{j,3} * MAVRE_{t-1} + \alpha_{j,4} * MDPRE_{t-1} + \alpha_{j,5} * DJAN_t + e_{j,t} \quad (11)$$

	Coefficients on Instrumental Variables						adj-R ²
	α_{ij}	α_{i1}	α_{i2}	α_{i3}	α_{i4}	α_{i5}	
Portfolio 1	0.0023** (3.107)	-0.1925** (-4.243)	-0.4464** (-2.907)	0.0338** (5.34)	-0.0676* (-1.777)	-0.004** (-3.25)	0.147
Portfolio 2	0.0026** (2.564)	-0.2409** (-3.809)	-0.5887** (-2.75)	0.0522** (5.192)	-0.0861 (-1.623)	-0.0032* (-1.884)	0.128
Portfolio 3	0.0037** (2.036)	-0.4088** (-3.646)	-1.1985** (-3.158)	0.0781** (4.992)	-0.0947 (-1.007)	0.0048 (1.567)	0.105
Portfolio 4	0.0015** (2.577)	-0.2539** (-6.978)	-0.188 (-1.526)	0.0202** (3.973)	-0.0876** (-2.871)	0.0022** (2.192)	0.172
Portfolio 5	0.002** (2.755)	-0.3487** (-7.713)	-0.2754* (-1.799)	0.0307** (4.86)	-0.1192** (-3.143)	0.0026** (2.125)	0.205
Portfolio 6	0.0036** (2.342)	-0.6669** (-7.165)	-0.901** (-2.86)	0.0644** (4.957)	-0.1915** (-2.453)	0.0164** (6.496)	0.247
Portfolio 7	0.0008 (1.348)	-0.3573** (-9.35)	-0.0993 (-0.767)	0.0183** (3.427)	-0.0667** (-2.081)	0.0042** (4.057)	0.236
Portfolio 8	0.0013* (1.954)	-0.3551** (-8.727)	-0.1502 (-1.091)	0.0253** (4.456)	-0.0958** (-2.808)	0.0046** (4.202)	0.233
Portfolio 9	0.001 (0.832)	-0.5861** (-8.149)	-0.2716 (-1.116)	0.0366** (3.643)	-0.1218** (-2.108)	0.0164** (8.408)	0.272

Note: Instrumental variables are a constant, the lagged difference in turnover for the market as a whole ($DMAVTO_{t-1}$), the lagged cross-sectional dispersion of turnover for the market as a whole ($MDPTO_{t-1}$), the lagged average market return ($MAVRE_{t-1}$), the lagged dispersion in returns for the market ($MDPRE_{t-1}$), and a January dummy variable ($DJAN_t$). A double (single) asterisk, ** (*), indicates significance at the .05 (.10) level.

Table 4

Chi-square Value and Significance Levels for Heteroskedasticity-Consistent Tests of Latent Variable Models that Test the Number of Factors with Turnover Data based on Turnover-Beta-Ranked Portfolios

Sampling Period	Number of Observations	<u>One-Factor Model</u>	<u>Two-Factor Model</u>	<u>Three-Factor Model</u>	<u>Four-Factor Model</u>
		Chi-Square (P-value)	Chi-Square (P-value)	Chi-Square (P-value)	Chi-Square (P-value)
03/1966 - 12/1976	130	26.02 (0.9571)	--	--	--
01/1977 - 12/1986	120	50.48 (0.1239)	--	--	--
01/1987 - 12/1996	120	53.67 (0.0728)	35.36 (0.1596)	--	--

Note: The models restrict the system of regression of J changes in turnovers of nine stratified equal-weighted portfolios on L instrumental variables. For each month from January 1962 to December 1996, securities are sorted into 3 size-ranked portfolios according to their previous year-end market capitalization. Each size portfolio is then further partitioned into 3 turnover-sorted portfolios according to the turnover betas of its component securities. The turnover of a stock in a month is defined as the total trading volume of the month divided by the total number of shares outstanding at the end of the previous month. The turnover of a portfolio is an equal-weighted average of individual turnovers. The monthly change in turnover is the first difference of the monthly portfolio turnover and that of the previous month. Instrumental variables are a constant, the lagged difference in turnover for the market as a whole ($DMAVTO_{t-1}$), the lagged cross-sectional dispersion of turnover for the market as a whole ($MDPTO_{t-1}$), the lagged average market return ($MAVRE_{t-1}$), the lagged dispersion in returns for the market ($MDPRE_{t-1}$), and a January dummy variable ($DJAN_t$). Define δ_{ij} as the coefficient of the i th portfolio's change in turnover on the j th instrumental variable. A K -latent-variable model imposes $(J - K)(L - K)$ restrictions on the system, which can be written as

$$\delta_{ij} = \sum_{k=1}^K \beta_{jk} \alpha_{ki}$$

Where β_{ik} and α_{kj} are free parameters. The tests are conducted by estimating the restricted system and computing Hansen's (1982) chi-square statistic.

Table 5

**Regressions of Portfolio Returns on Instrumental Variables
over the Period 1966 to 1996**

$$PRET_{j,t} = \alpha_{j,0} + \alpha_{j,1} * DPR_{t-1} + \alpha_{j,2} * LTMST_{t-1} + \alpha_{j,3} * RSR_{t-1} + \alpha_{j,4} * DMAVTO_{t-1} + \alpha_{j,5} * DJAN_t + e_{j,t} \quad (12)$$

	Coefficients on Instrumental Variables						Adj-R ²
	α_{i0}	α_{i1}	α_{i2}	α_{i3}	α_{i4}	α_{i5}	
Portfolio 1	0.0081* (1.862)	0.9934 (0.908)	0.2063** (2.077)	-5.7301** (-2.249)	1.0981*** (2.845)	0.0872*** (7.637)	0.182
Portfolio 2	0.0064 (1.286)	1.3098 (1.051)	0.1682 (1.487)	-7.876*** (-2.713)	1.2709*** (2.891)	0.0922*** (7.085)	0.165
Portfolio 3	0.0021 (0.361)	1.44 (1.006)	0.197 (1.517)	-8.0766** (-2.424)	1.2775** (2.531)	0.0976*** (6.533)	0.141
Portfolio 4	0.0055 (1.587)	1.4041 (1.62)	0.2348*** (0.985)	-3.2868 (-1.628)	0.3723 (1.218)	0.0322*** (3.562)	0.064
Portfolio 5	0.0054 (1.303)	1.5779 (1.515)	0.2764*** (2.925)	-4.86** (-2.004)	0.4938 (1.345)	0.0379*** (3.487)	0.066
Portfolio 6	0.0005 (0.104)	2.1523* (1.661)	0.2155* (1.833)	-6.5663** (-2.176)	0.6103 (1.335)	0.0492*** (3.64)	0.056
Portfolio 7	0.0056* (1.727)	1.3424 (1.647)	0.182** (2.461)	-2.98 (-1.57)	-0.0739 (-0.257)	0.0155** (1.828)	0.024
Portfolio 8	0.0046 (1.347)	1.5212* (1.783)	0.2144*** (2.77)	-3.277 (-1.65)	-0.0879 (-0.292)	0.0182** (2.04)	0.031
Portfolio 9	0.0036 (0.885)	1.5821 (1.534)	0.2266** (2.421)	-4.687* (-1.952)	0.1053 (0.29)	0.0236** (2.19)	0.031

Note: The independent variables are a constant, the lagged dividend yield on the market (DPR_{t-1}), the lagged spread between returns on the 20-year government bond and the one-month T-bill ($LTMST_{t-1}$), the lagged relative short rate (RSR_{t-1}), i.e., the 30-day T-bill total return minus the moving average total return on 30-day T-bills over the last year, the lagged change in average turnover for the whole market ($DMAVTO_{t-1}$), and a January dummy variable ($DJAN_t$). A double (single) asterisk, ** (*), indicates significance at the .05 (.10) level.

Table 6

Chi-square Value and Significance Levels for Heteroskedasticity-Consistent Tests of Latent Variable Models that Test the Number of Factors with Return Data based on Turnover-Beta-Ranked Portfolios

Sampling Period	Number of Observations	<u>One-Factor Model</u>	<u>Two-Factor Model</u>	<u>Three-Factor Model</u>	<u>Four-Factor Model</u>
		Chi-Square (P-value)	Chi-Square (P-value)	Chi-Square (P-value)	Chi-Square (P-value)
03/1966 - 12/1976	130	47.90 (0.1828)	--	--	--
01/1977 - 12/1986	120	49.94 (0.1349)	--	--	--
01/1987 - 12/1996	120	49.05 (0.1545)	--	--	--

Note: The models restrict the system of regressions of J returns of nine stratified equal-weighted portfolios on L instrumental variables. For each month from January 1962 to December 1996, securities are sorted into 3 size-ranked portfolios according to their previous year-end market capitalization. Each size portfolio is then further partitioned into 3 turnover-sorted portfolios according to the turnover betas of its component securities. The turnover of a stock in a month is defined as the total trading volume of the month divided by the total number of shares outstanding at the end of the previous month. The turnover of a portfolio is an equal-weighted average of individual turnovers. The monthly change in turnover is the first difference of the monthly portfolio turnover and that of the previous month. The instrumental variables are a constant, the lagged dividend yield on the market (DPR_{t-1}), the lagged spread between returns on the 20-year government bond and the one-month T-bill ($LTMST_{t-1}$), the lagged relative short rate (RSR_{t-1}), i.e., the 30-day T-bill total return minus the moving average total return on 30-day T-bills over the last year, the lagged change in average turnover for the whole market ($DMAVTO_{t-1}$), and a January dummy variable ($DJAN_t$). Define δ_{ij} as the coefficient of the i th portfolio's change in turnover on the j th instrumental variable. A K -latent-variable model imposes $(J - K)(L - K)$ restrictions on the system, which can be written as

$$\delta_{ji} = \sum_{k=1}^K \beta_{jk} \alpha_{ki}$$

Where β_{ik} and α_{kj} are free parameters. The tests are conducted by estimating the restricted system and computing Hansen's (1982) chi-square statistic.

Table 7

Chi-square Value and Significance Levels for Heteroskedasticity-Consistent Tests of Latent Variable Models that Test the Number of Factors with Turnover and Return Data based on Return-Beta-Ranked Portfolios

Sampling Period	Number of Observations	<u>One-Factor Model</u>	<u>Two-Factor Model</u>	<u>Three-Factor Model</u>	<u>Four-Factor Model</u>
		Chi-Square (P-value)	Chi-Square (P-value)	Chi-Square (P-value)	Chi-Square (P-value)
Panel A: Turnover					
03/1966 - 12/1976	130	38.18 (0.5524)	--	--	--
01/1977 - 12/1986	120	44.54 (0.2826)	--	--	--
01/1987 - 12/1996	120	46.05 (0.2326)	--	--	--
Panel B: Returns					
03/1966 - 12/1976	130	42.29 (0.3725)	--	--	--
01/1977 - 12/1986	120	65.37 (0.0069)	37.53 (0.1077)	--	--
01/1987 - 12/1996	120	52.37 (0.0910)	36.71 (0.1254)	--	--

Note: The models restrict the system of regressions of J returns of nine stratified equal-weighted portfolios on L instrumental variables. For each month from January 1962 to December 1996, securities are sorted into 3 size-ranked portfolios according to their previous year-end market capitalization. Each size portfolio is then further partitioned into 3 turnover-sorted portfolios according to the turnover betas of its component securities. The turnover of a stock in a month is defined as the total trading volume of the month divided by the total number of shares outstanding at the end of the previous month. The turnover of a portfolio is an equal-weighted average of individual turnovers. The monthly change in turnover is the first difference of the monthly portfolio turnover and that of the previous month. The instrumental variables for the change in turnover and returns are the same as those described in Tables 4 and 6. Define δ_{ij} as the coefficient of the i th portfolio's change in turnover on the j th instrumental variable. A K -latent-variable model imposes $(J - K)(L - K)$ restrictions on the system, which can be written as

$$\delta_{ij} = \sum_{k=1}^K \beta_{jk} \alpha_{ki}$$

Where β_{ik} and α_{kj} are free parameters. The tests are conducted by estimating the restricted system and computing Hansen's (1982) chi-square statistic.