Efficiency wages and futures markets

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Keywords: Efficiency wages; Price uncertainty; Futures markets

JEL classification: D21; J31

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This paper places the competitive firm *à la* Sandmo in a standard efficiency wage model, wherein the work effort of labor depends on the wage rate set by the firm. Irrespective of the availability of hedging opportunities, we show that the Solow condition under which the equilibrium effort-wage elasticity equals unity is a norm. Thus, the optimal wage rate paid by the firm is invariant to the risk attitude of the firm and to the incidence of output price uncertainty. We further show that hedging activities induce the firm to hire more labor as a result of the reduction in its risk exposure. Thus, the introduction of futures markets is beneficial in that employment is increased and output is enhanced.

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1. Introduction

The use of financial derivatives to assess and manage exposures to various sources of risk is a norm rather than an exception in modern corporations. A 1995 *Fortune* article summarizes the characteristics of financial derivatives: "These financial innovations both warm and burn.... Love them or hate them, they're all here to stay." As financial derivatives are here to stay, corporate managers have to understand them and know how to use them to reduce risk in an effective way.

Given the real-world prominence of risk management, there have been a great many papers concerning the production and hedging decisions of a risk-averse firm under uncertainty (see, e.g., Danthine, 1978; Holthausen, 1979; Katz and Paroush, 1979; Kawai and Zilcha, 1986; Broll, Wong, and Zilcha, 1999; Broll, Chow, and Wong, 2000). Two notable results have emanated from the literature. First, the "separation theorem" states that the production decision of the firm is affected neither by the risk attitude of the firm nor by the incidence of the underlying uncertainty should the firm have access to a futures market.[1] Second, the "full-hedging theorem" states that the firm should eliminate all risks by holding a full hedge if the futures market is unbiased.

The purpose of this paper is to re-examine the optimality of the separation and full-hedging theorems in a standard efficiency wage model. The essential feature of the efficiency wage hypothesis is that wages enter into the production function of the firm in a labor-augmenting way. The positive dependence of work effort (or productivity) on wages can be justified on the grounds of moral hazard, turnover costs, adverse selection, and/or sociological reasons (see, e.g., Solow, 1979, 1990; Schlicht, 1978, 1992; Salop, 1979; Wesis, 1980; Akerlof, 1982; Yellen, 1984; Shapiro and Stiglitz, 1984). Efficiency wages differ from market clearing wages in that the latter serve the function of equating demand and supply in the labor market while the former prevent workers from leaving or shirking. In this sense, efficiency wages are devised so as to control future behavior rather than past behavior and thus may be regarded as forward-looking (Schlicht, 1995). As advocated by Blanchard and Fischer (1986), "efficiency wage theory is surely one of the most promising directions of research at this stage."

In this paper, we present the competitive firm facing output price uncertainty

[1] This is a rather surprising result in light of the seminal works of Baron (1970) and Sandmo (1971) which show that production decisions are affected by risk factors and preferences in the absence of hedging opportunities.

à la Sandmo (1979), placed in a standard efficiency wage model wherein the work effort of labor depends on the wage rate set by the firm. We show that both the separation and full-hedging theorems remain valid in this setting. Furthermore, the well-known result that the equilibrium effort-wage elasticity is unity, or the so-called "Solow condition," holds irrespective of the availability of hedging opportunities to the firm.[2] As a result, the optimal wage rate paid by the firm is invariant to the risk attitude of the firm and to the incidence of output price uncertainty. Finally, we show that hedging activities induce the firm to hire more labor as a result of the reduction in its risk exposure. In other words, the introduction of futures markets benefits society in that employment is increased and output is enhanced.

The rest of the paper is organized as follows. The next section develops a model of the competitive firm under output price uncertainty and the efficiency wage hypothesis. Section 3 characterizes the optimal production and hedging decisions of the firm when a commodity futures market is present. Section 4 examines the economic implications of hedging with the commodity futures market on employment and productivity. The final section offers some concluding remarks.

2. The model

Consider a competitive firm which operates for one period with two dates, 0 and 1. Initially, the firm produces a single output, Q, using labor, L, as the sole input. According to Malcomson (1981), the essential feature of the efficiency wage hypothesis is that the productivity of labor increases when wages rise. As such, wages enter into

[2] The Solow condition is named by Akerlof and Yellen (1986).

the firm's production function in the following labor-augmenting way:

$$Q = F[e(w)L],$$

where $e(w)$ is the effort or productivity per worker, depending on the wage rate, w, paid by the firm. We assume that e is twicely continuously differentiable with $e' > 0$ and $e'' < 0$.[3] Define $\lambda = e(w)L$ as labor input measured in efficiency units. The production function is twicely continuously differentiable with $F' > 0$ and $F'' < 0$.

At date 1, the firm sells its entire output at the then prevailing output price, $\tilde{P}$, which is a positive random variable with a known probability distribution function.[4] Since the firm does not know ex-ante the ex-post realization of $\tilde{P}$, it inevitably exposes itself to output price uncertainty. The firm, however, has access to a commodity futures market wherein it can sell (purchase if negative) H units of its output forward at a pre-specified futures price, P_0, at date 0. Thus, the date 1 profit of the firm is given by

$$\tilde{\Pi} = [\tilde{P}F(\lambda) - wL] + (P_0 - \tilde{P})H, \tag{1}$$

where the first term in the right-hand side of equation (1) is the firm's operating profit, and the second term is the net gain or loss to the firm from its position in the commodity futures market.

The firm possesses a von Neumann-Morgenstern utility function, $U(\Pi)$, defined over its date 1 profit, Π. U is twicely continuously differentiable with $U' > 0$ and $U'' < 0$, indicating the presence of risk aversion. Before any uncertainty is resolved,

[3]See Pisauro (1991) for the derivation of an effort supply function possessing these properties from a moral hazard model of expected utility maximizing workers.

[4]Throughout the paper, a tilde (~) always signifies a random variable.

the firm chooses an employment level, L, a wage rate, w, and a futures position, H, so as to maximize the expected utility of its date 1 profit:

$$\max_{L,\ w,\ H}\ \mathrm{E}[U(\tilde{\Pi})], \tag{2}$$

where E is the expectation operator and $\tilde{\Pi}$ is defined in equation (1).

3. Optimal production and hedging decisions

The first-order conditions for a maximum of program (2) are given by

$$\mathrm{E}\{U'(\tilde{\Pi}^*)[\tilde{P}F'(\lambda^*)e(w^*) - w^*]\} = 0, \tag{3}$$

$$\mathrm{E}\{U'(\tilde{\Pi}^*)[\tilde{P}F'(\lambda^*)e'(w^*) - 1]\}L^* = 0, \tag{4}$$

$$\mathrm{E}[U'(\tilde{\Pi}^*)(P_0 - \tilde{P})] = 0, \tag{5}$$

where an asterisk (*) indicates an optimal level, and $\lambda^* = e(w^*)L^*$ is the optimal labor input in efficiency units. The second-order conditions are assumed to be satisfied.

Multiplying w^*/L^* to equation (4) and substituting the resulting equation to equation (3) yields

$$\mathrm{E}[U'(\tilde{\Pi}^*)\tilde{P}]F'(\lambda^*)[e(w^*) - e'(w^*)w^*] = 0.$$

Since U', $\tilde{P}$, and F' are all strictly positive, the above equation holds if, and only if,

$$\frac{e'(w^*)w^*}{e(w^*)} = 1. \tag{6}$$

In words, equation (6) says that the optimal wage rate, w^*, is attained when the cost of one efficiency unit of labor, w/e, is minimized. This is the well-known Solow condition in the efficiency wage literature.[5] The following proposition is invoked.

Proposition 1. *In the presence of the commodity futures market, the Solow condition under which the equilibrium effort-wage elasticity is unity holds.*

Substituting equation (5) into equation (3) yields

$$\mathrm{E}[U'(\tilde{\Pi}^*)][P_0 F'(\lambda^*)e(w^*) - w^*] = 0.$$

Since $U' > 0$, the above equation holds if, and only if,

$$P_0 F'(\lambda^*)e(w^*) = w^*. \tag{7}$$

Equation (7) implies that, at the optimum, the marginal product of labor equals the wage rate, where the random output price is locked in at the known futures price, P_0. An immediate implication of equations (6) and (7) is the following proposition which is the celebrated separation theorem in the hedging literature.

Proposition 2. *In the presence of the commodity futures market, the firm's optimal employment and wage offer depend neither on its attitude towards risk nor on the incidence of the output price uncertainty.*

To see the intuition of Proposition 2, we rewrite the firm's date 1 profit as

$$\tilde{\Pi} = [P_0 F(\lambda) - wL] + (P_0 - \tilde{P})[H - F(\lambda)].$$

[5]See Schmidt-Sørensen (1990) and Lin and Lai (1994, 1997) for interesting examples where the equilibrium effort-wage elasticity may be less than unity, thereby invalidating the Solow condition.

The first term in the right-hand side of the above equation is the firm's operating profit when the random output price is locked in at the pre-specified futures price, P_0. The second term is the gain or loss due to the firm's position in the commodity futures market being deviated from a full hedge, i.e., $H = F(\lambda)$. Thus, the firm's exposure to the output price uncertainty is entirely controlled by its futures position and is totally unrelated to its production decision, thereby invoking the separation theorem.

Using the covariance operator, Cov, equation (5) can be written as[6]

$$\mathrm{E}[U'(\tilde{\Pi}^*)][P_0 - \mathrm{E}(\tilde{P})] = \mathrm{Cov}[U'(\tilde{\Pi}^*), \tilde{P}]. \tag{8}$$

Since $\tilde{\Pi}^* = [P_0 F(\lambda^*) - w^* L^*] + (P_0 - \tilde{P})[H^* - F(\lambda^*)]$, the realization of $\tilde{\Pi}^*$ is increasing with, invariant to, or decreasing with the realization of $\tilde{P}$ depending on whether H^* is less than, equal to, or greater than $F(\lambda^*)$. If the commodity futures market is unbiased so that the pre-specified futures price, P_0, equals the expected date 1 price, $\mathrm{E}(\tilde{P})$, equation (8) implies a full coverage of the output price risk exposure, i.e., $H^* = F(\lambda^*)$. This is the famous full-hedging theorem in the hedging literature.

The intuition of the full-hedging theorem is that the unbiased commodity futures market essentially provides the firm 'insurance' at actuarial terms, rendering a full hedge by the firm optimal. If the commodity futures market exhibits contango so that $P_0 > \mathrm{E}(\tilde{P})$, the firm will speculate by holding an over hedge, i.e., $H^* > F(\lambda^*)$, hoping to gain from a lower date 1 price at expiration. Finally, if the commodity futures market exhibits normal backwardation so that $P_0 < \mathrm{E}(\tilde{P})$, the firm will speculate by adopting an under hedge, i.e., $H^* < F(\lambda^*)$, expecting to gain from a higher date 1 price at expiration.

[6]For any two random variables, $\tilde{x}$ and $\tilde{y}$, $\mathrm{Cov}(\tilde{x}, \tilde{y}) = \mathrm{E}(\tilde{x}\tilde{y}) - \mathrm{E}(\tilde{x})\mathrm{E}(\tilde{y})$.

To summarize, we have the following proposition.

Proposition 3. *In the presence of the commodity futures market, the firm's optimal futures position is an over hedge, a full hedge, or an under hedge, depending on whether the commodity futures market exhibits contango, unbiasedness, or normal backwardation, respectively.*

4. Effects of hedging on employment and productivity

It is of interest to see what role the commodity futures market plays in the firm's optimal decisions. To this end, we consider a benchmark case in which the commodity futures market is either absent or is not accessible by the firm. In either case, we have $H \equiv 0$. Thus, the firm's decision problem becomes

$$\max_{L,\ w} \ \mathrm{E}[U(\tilde{\Pi})] \ \text{ s.t. } \ H \equiv 0, \tag{9}$$

where $\tilde{\Pi}$ is defined in equation (1). The first-order conditions for a maximum of program (9) are given by

$$\mathrm{E}\{U'(\tilde{\Pi}^0)[\tilde{P}F'(\lambda^0)e(w^0) - w^0]\} = 0, \tag{10}$$

$$\mathrm{E}\{U'(\tilde{\Pi}^0)[\tilde{P}F'(\lambda^0)e'(w^0) - 1]\}L^0 = 0, \tag{11}$$

where a nought (^{0}) indicates an optimal level, and $\lambda^0 = e(w^0)L^0$ is the optimal labor input in efficiency units. The second-order conditions are assumed to be satisfied.

Multiplying w^0/L^0 to equation (11) and substituting the resulting equation to equation (10) yields

$$\mathrm{E}[U'(\tilde{\Pi}^0)\tilde{P}]F'(\lambda^0)[e(w^0) - e'(w^0)w^0] = 0.$$

Since U', $\tilde{P}$, and F' are all strictly positive, the above equation holds if, and only if,

$$\frac{e'(w^0)w^0}{e(w^0)} = 1,$$

which is simply the Solow condition. Using Proposition 1, we establish the following result.

Proposition 4. *Irrespective of the presence or absence of the commodity futures market, the Solow condition under which the equilibrium effort-wage elasticity is unity always holds.*

An immediate implication of Proposition 4 is that the optimal wage rate paid by the firm is invariant to the risk attitude of the firm and to the incidence of the output price uncertainty. This result is rather intuitive because the optimal wage rate minimizes the cost of one efficiency unit of labor which depends only on the production technology.

Rewrite equation (10) as

$$\mathrm{E}[U'(\tilde{\Pi}^0)(\tilde{P} - P_0)]F'(\lambda^0)e(w^0) + \mathrm{E}[U'(\tilde{\Pi}^0)][P_0F'(\lambda^0)e(w^0) - w^0] = 0, \tag{12}$$

where P_0 is the futures price. The first term in the left-hand side of equation (12) can be written as

$$\mathrm{Cov}[U'(\tilde{\Pi}^0), \tilde{P}]F'(\lambda^0)e(w^0) + \mathrm{E}[U'(\tilde{\Pi}^0)][\mathrm{E}(\tilde{P}) - P_0]F'(\lambda^0)e(w^0).$$

Since $F' > 0$ and $e > 0$, the above expression is negative if, and only if,

$$P_0 > \mathrm{E}(\tilde{P}) + \frac{\mathrm{Cov}[U'(\tilde{\Pi}^0), \tilde{P}]}{\mathrm{E}[U'(\tilde{\Pi}^0)]}. \tag{13}$$

If this condition holds (i.e., the futures price is sufficiently high), then equation (12) implies that

$$P_0 F'(\lambda^0) e(w^0) > w^0. \tag{14}$$

From Proposition 4, we have $w^* = w^0$. Since $F'' < 0$, it follows from equations (7) and (14) that $L^0 < L^*$ whenever equation (13) holds. Hence, we establish the following proposition.

Proposition 5. *The introduction of the commodity futures market with a sufficiently high futures price induces the firm to hire more labor and thereby produce more output.*

Note that $\mathrm{Cov}[U'(\tilde{\Pi}^0), \tilde{P}] < 0$ by risk aversion (i.e., $U'' < 0$). A sufficient, albeit not necessary, condition for equation (13) to hold is that the commodity futures market does not exhibit normal backwardation. In this case, we can conclude that the introduction of the commodity futures market is beneficial in that employment is increased and output is enhanced.

5. Concluding remarks

Efficiency wage models differ from standard labor market models in that wages do not necessarily clear markets. Wages are viewed as devices to encourage work effort or, more generally, to increase productivity. Firms optimally set their wages above the market clearing level, thereby generating involuntary unemployment. This paper has employed a standard efficiency wage model to address the optimal efficiency wage and allocation decisions for the competitive firm under output price uncertainty *à la* Sandmo (1971).

We have shown that the celebrated separation and full-hedging theorems derived in the hedging literature hold in our setting. Furthermore, the Solow condition under which the equilibrium effort-wage elasticity equals unity is a norm no matter whether hedging opportunities are available to the firm or not. An immediate implication is that the optimal wage rate paid by the firm depends neither on its attitude towards risk nor on the incidence of the output price uncertainty. Finally, we have shown that the firm hires more labor if it engages in hedging activities. Thus, both employment and output levels are enhanced in the presence of risk-sharing futures markets.

References

Akerlof, G.A., 1982, Labor contracts as a partial gift exchange, Quarterly Journal of Economics 97, 543–569.

Akerlof, G.A. and J.L. Yellen, 1986, Introduction, in: G.A. Akerlof and J.L. Yellen, eds., Efficiency Wage Models of the Labor Market, Cambridge: Cambridge University Press.

Baron, D., 1970, Price uncertainty, utility and industry equilibrium in pure competition, International Economic Review 11, 463–480.

Blanchard, O.J. and S. Fischer, 1989, Lectures on Macroeconomics, Cambridge, MA:

MIT Press.

Broll, U., K.W. Chow and K.P. Wong, 2000, Hedging and nonlinear risk exposure, Oxford Economic Papers, forthcoming.

Broll, U., K.P. Wong and I. Zilcha, 1999, Multiple currencies and hedging, Economica 66, 421–432.

Danthine, J.-P., 1978, Information, futures prices, and stabilizing speculation, Journal of Economic Theory 19, 79–89.

Holthausen, D.M., 1979, Hedging and the competitive firm under price uncertainty, American Economic Review 69, 989–995.

Katz, E. and J. Paroush, 1979, The effect of forward markets on exporting firms, Economics Letters 4, 272–274.

Lin, C.-C. and C.-C. Lai, 1994, The turnover costs and the Solow condition in an efficiency wage model with intertemporal optimization, Economics Letters 45, 501–505.

Lin, C.-C. and C.-C. Lai, 1997, The Solow condition in an efficiency wage model with overtime work, Australian Economic Papers 36, 342–350.

Malcomson, J.M., 1981, Unemployment and the efficiency wage hypothesis, Economic Journal 91, 848–866.

Pisauro, G., 1991, The effect of taxes on labour in efficiency wage models, Journal of Public Economics 46, 329–345.

Salop, S., 1979, A model of the natural rate of unemployment, American Economic Review 69, 17–25.

Sandmo, A., 1971, On the theory of the competitive firm under price uncertainty, American Economic Review 61, 65–73.

Schlicht, E., 1978, Labor turnover, wage structure and natural unemployment, Journal of Institutional and Theoretical Economics 134, 337–346.

Schlicht, E., 1992, Wage generosity, Journal of Institutional and Theoretical Economics 148, 437–451.

Schlicht, E., 1995, Autonomous wage inflation, in: Determinanten der Lohnbildung, in: K. Gerlach and R. Schettkat, eds., Berlin, edition sigma, 187–205.

Schmidt-Sørensen, J.B., 1990, The equilibrium effort-wage elasticity in efficiency-wage models, Economics Letters 32, 365–369.

Shapiro, C. and J.E. Stiglitz, 1984, Equilibrium unemployment as a worker discipline device, American Economic Review 74, 433–444.

Solow, R.M., 1979, Another possible source of wage stickiness, Journal of Macroeconomics 1, 79–82.

Solow, R.M., 1990, The Labor Market as a Social Institution, Cambridge, Mass.: Basil Blackwell.

Weiss, A., 1980, Job queues and layoffs in labor markets with flexible wages, Journal of Political Economy 88, 526–538.

Yellen, J.L., 1984, Efficiency wage models of unemployment, American Economic Review Papers and Proceedings 74, 200–205.